Optimization of Inventory Distribution Logistics in Industrial Enterprises

Optimización de la logística de distribución de inventario en empresas industriales

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ABSTRACT:
The paper studies the methods for solving the problem of optimizing logistics for the distribution of reserves in industrial enterprises. The analysis of the known basic models led to the conclusion on their limited prognostic potential, which does not allow considering the business cycles and different types of stocks by the shelf life. This paper investigates the reasons for low efficiency of the basic models of warehouse logistics. In solving this problem, the authors propose a modified approach with the calculation of the order size for the formation of reserves at the optimum level of compliance with the production needs. The terms have been identified for application of the model with the cost of order, depending on the batch size. The approach was approbated via simulation modeling for different groups of reserves by functional criteria among enterprises of one industry. The model of the fair value of production reserves was chosen as the resultant model. The results of approbation testify to the adequacy of the model with a high reliability of evaluation.

Keywords: logistic function, management model, production optimization, stock distribution

RESUMEN:
El documento estudia los métodos para resolver el problema de optimizar la logística para la distribución de reservas en empresas industriales. El análisis de los modelos básicos conocidos llevó a la conclusión sobre su potencial pronóstico limitado, lo que no permite considerar los ciclos económicos y los diferentes tipos de existencias por la vida útil. Este documento investiga las razones de la baja eficiencia de los modelos básicos de logística de almacén. Para resolver este problema, los autores proponen un enfoque modificado con el cálculo del tamaño del pedido para la formación de reservas en el nivel óptimo de cumplimiento con las necesidades de producción. Los términos han sido identificados para la aplicación del modelo con el costo de pedido, dependiendo del tamaño del lote. El enfoque fue aprobado a través de modelos de simulación para diferentes grupos de reservas por criterios funcionales entre las empresas de una industria. El modelo del valor razonable de las reservas de producción se eligió como el modelo resultante. Los resultados de la aprobación atestiguan la idoneidad del modelo con una alta fiabilidad de la evaluación.

Palabras clave función logística, modelo de gestión, optimización de la producción, distribución de existencias
1. Introduction

Companies, taking into account their resource potential, rely on the possibility of planning sustainable development. One of the most effective tools for managing the movement of material, financial, information and other resources in production and turnover is logistics (Katotkov, 2005). Logistics is a costly part of an enterprise’s business processes. In general, logistics costs are caused by the movement of material flows. Logistic processes of an enterprise have a comprehensive structure, a non-linear relationship and are influenced by many factors. For their description, a large number of parameters and a large array of information are required.

The basis of successful operation of any enterprise is the solution of the problem of minimizing costs in inventory management and at the same time maintaining the rhythm of production without stopping. Finding a balance between these two contradictions is the main goal of optimal inventory management.

Among industrial enterprises, only a few have a well-developed logistics strategy that provides for integrated management of material flows, taking into account modern life-cycle concepts (Kuzmin, 2017). In the modeling of logistics strategies, such criteria as the reduction of total costs (Ivanov et al., 2017; Nechaeva et al., 2016), differentiation of logistics services (Burinskas, & Burinskiene, 2016; Scarsi, & Spinelli, 2017; Ivanova, 2017), innovation (Klapalová, 2013) and others are used. The literature review points to a wide variety of forms and conditions regarding the level of logistic management, which does not allow one to unambiguously interpret the types of logistic strategies.

As a result, a whole complex of mathematical problems arises, the solution of which allows creating a stable logistic system (Lubentsova, 2008). Consequently, the construction of mathematical models of inventory management with a high degree of adequacy is an actual problem when creating an effective logistics system. Nevertheless, some theoretical and practical aspects are still poorly studied.

2. Literature review

A common stock management approach is the Wilson model (Wilson, 1934) (it is also called a deterministic model for a system with a fixed order size). This model describes the process of inventory management and is characterized by the following assumptions: the intensity of the use of reserves is a priori known and constant; the delivery time is constant and known; costs do not depend on the size of the order; the absence of stocks is unacceptable. The peculiarity of this model is that the change in the level of stocks is cyclical and all the cycles of change in stocks are the same, and the maximum number of products in stock coincides with the size of the order Q. However, in practice, the demand size can almost never be specified accurately: most often, it is described in probabilistic quantities over a range interval. According to Sterligova, "it can be stated that the instrumentation in question (including all modifications of the Wilson formula) has a negative reputation among specialists. It is considered purely theoretical, unacceptable for practice; and the result of the calculation has a significant deviation from the accepted batches of orders" (Sterligova, 2005).

Based on the Wilson model, a set of inventory management models was created for various options of the functioning of logistical processes. For example, the model taking into account the change in supply costs, the model taking into account the uneven execution time of the order and the demand for the material (Solyanik, 2006), the model with VAT (Sterligova, & Semenova, 2005), etc. It seems that a deeper study of the problem is possible only if the models are classified by the complexity of their application and qualitatively new conditions in designing for different periods (Tektov, 2003).

To solve this problem, a number of researchers propose modified models. These complexities of operational inventory management are solved in the model \(<Q, r>\) – a model taking into account the requirements unmet, characterized by the following assumptions: the cost of a unit of inventory does not depend on the batch size; in the
system there is not more than one uncompleted order; costs do not depend on $Q_i$ of the order level $r$. Unlike deterministic models, in this model, during each cycle, the system may not retain the accuracy of the order time and the cycle itself is now random (with the fictitious reserve level varying from $r$ to $r + Q$).

The main problem arising when applying this model is that the state of the system at any time is unknown. The moments when requirements arise (or the size of the requirements) are random. Therefore, in order to monitor the system at any time, all operations and transactions (requirements, filing orders, receipt of goods) must be immediately registered, which is often impossible in practice.

Taking into account the mentioned shortcomings, the models considered above have a low prognostic potential. They cannot be applied to manufacturing using products that have a limited shelf life, or when it is not possible to determine the exact value of demand for raw materials and to fix the state of the system at any given time. To solve this, it is necessary to identify and eliminate the reasons for the weak adequacy of mathematical models of inventory management.

3. Methods
Increasing the efficiency of optimal inventory management occurs due to the use of a dynamic model that takes into account the costs on organizing and servicing stocks. Let us construct a mathematical model for the next task of optimizing the inventory management scheme.

Let the warehouse have $n$ types of products. Each unit of the $i$-th product, $i = 1, \ldots, n$, has a fixed shelf life of $m$. The stock level is constantly monitored by meeting demand or eliminating stock units. All order units (requirements) for replenishment are new. The utility of each unit of the product does not decrease and does not disappear until the expiration of the storage period, but the product must be noted, if it was not used before the expiration of the storage period. The costs incurred due to the aging of the $i$-th product are equal to $W_i$ per unit. The demand for the $i$-th product per unit of time is $d_i$. Each unit of product of the $i$-th type occupies a volume in the storage room equal to $V_i$. The entire volume of the warehouse is $V$. The stock units are always used in accordance with the FIFO policy (the first unit that got into the composition is also used first). In each period for each product, there is a limit on the size of the order: one can put no more than $Z_i$ units of product. Also, the following indicators are introduced: $C_i$ – the cost of replenishing the unit of stock of the $i$-th product; $h_i$ – the cost of maintaining the unit of stock of the $i$-th product in a unit of time; $P_i$ – costs associated with accounting for unsatisfied demand (per unit); $\vartheta_i$ – costs associated with losses of unsatisfied demand (per unit); $\beta_i$ – the share of unsatisfied demand in the replenishment cycle.
costs associated with losses or unsatisfied demand (per unit); \( \beta \) - the share of unsatisfied demand in the replenishment cycle, which can be a debt, and \((1-\beta)\) is the lost share.

It is necessary to find the size of orders \( q^* \) for each type of product at each moment of time at which the minimum costs for storage of products over the period \( T \) are reached. Based on the condition of the task, the authors formulated a list of indicators that affect the value of the objective function, these including: the cost of replenishment of stock; costs for storage of products in the warehouse; costs associated with a shortage of products; costs associated with obsolescence of products.

Given all of the above, one can conclude that the objective function of the optimization problem is the sum of all costs for all types of products over the entire period of time:

\[
g(q_{t,j}) = R = \sum_{i=1}^{T} \sum_{j=1}^{n} \left[ q_{t,j} \times C_i + P_i \times \beta \times \left( d_{t,j} - U_{t,j} \right) + \theta_i \times (1-\beta) \times \left( d_{t,j} - U_{t,j} \right)^+ \right. \\
+ W_i \times \left( x_{t,i,j} - d_{t,j} \right)^+ + h_i \times \left( U_{t,i} - d_{t,j} \right)^+ \right]
\]

Hence, the multi-product model of stock management in a warehouse with a limited volume for products with a limited shelf life will have the following form:

\[
R \rightarrow \min
\]

with restrictions:

\[
\begin{align*}
0 & \leq q_{t,i} \leq Z_{t,i}, \\
\sum_{i=1}^{n} V_i \times U_{t,i} & \leq V
\end{align*}
\]

where \( Z_{t,i} \) - the maximum size of the order for replenishment of the \( i \)-th type of product at the time \( t \); \( U_{t,i} \) - the level of stock of the \( i \)-th product at the time \( t \).

The value of the stock level of the \( i \)-th product is determined by the following formulae:

\[
U_{t,i} = \sum_{j=1}^{m_i} x_{t,i,j}
\]

\[
x_{t,i,j} = \begin{cases} \left[ q_{t,i} \right]^+, \text{ where } j = m_i \\
\left[ x_{t-1,i,j+1} - \left( d_{t,i} - \sum_{i=1}^{j} x_{t-1,i,j} \right)^+ \right]^+, \text{ where } j < m_i
\end{cases}
\]

where \( d_{t,i} \) - demand for the \( i \)-th type of product at the time \( t \); \( m_i \) - shelf life of the \( i \)-th type of product; \( V, u_j \) and \( V \) - constants defined; \( q_{t,i} \) - the size of the order to replenish the stock of the \( i \)-th type of product, which must be done at the time \( t \);

\[
[a]^+ = \max [0, a].
\]

Since all possible order values are a bounded set and are expressed as integers, here is a problem that belongs to the class of discrete nonlinear programming tasks. To solve the problem, the authors used approximate methods of combinatorial optimization, the most developed at the moment being the methods of local optimization (the method of the recession vector (Sergienko et al., 1980), the simulated annealing method (Kirkpatrick et al., 1983; Ingber, 1993) and the accelerated probabilistic simulation algorithm (Groysberg, & Nesteruk, 1995)).

The next step is to identify the economic efficiency of optimizing inventory levels. The solution can be found in the context of 3 different models, which make it possible to calculate the reserves of not only the finished goods, but more importantly, at the design stage (before the formation of the actual reserves):

**Model 1 (M1)** provides for the calculation of the initial cost (historical cost) of the enterprise's production reserves:

\[
Y = x_1 + x_2 + x_3 + (x_{41} + x_{42} + x_{43}) = \sum_{j=1}^{3} x_j + \sum_{k=1}^{3} x_{4k}
\]

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**4. Results and Discussion**
5. Conclusions

The urgency and insufficient knowledge of the problems of accounting for the interaction of the main flowing processes in industrial enterprises determined the need for scientific developments in optimizing the logistics function. In this paper, some problems of low adequacy of models are considered. The result of the study was the modified model, in which the optimal size of demand was determined during the formation of stocks. To solve the problem of managing production reserves, a methodical approach was suggested that makes it possible to determine in a timely manner the normative level of industrial reserves via: Model 1 (M1) – realizing the principle of calculating the initial (historical) cost of production reserves; Model 2 (M2) – providing an estimate of production stocks as of the reporting date; Model 3 (M3) – designed to establish the fair value of inventories. The initial data for simulation modeling are presented in Annex with a total of observations in up to 100 enterprises. Correlation-regression models are constructed for various groups of production stocks: primary raw materials, auxiliary raw materials, raw materials for packing and forming, physical facilities. As an example, the enterprises of a particular industry (wood processing) with identical inquiries during the formation of industrial stocks were taken. As a result of the implementation of the Regression module in MS Excel, some models were obtained, the analytical form of being presented in Table 5.

Table 5 – Correlation-regression models of the cost of production stocks of enterprises by groups of stocks

<table>
<thead>
<tr>
<th>Group of material stocks</th>
<th>Model’s analytical view</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1 (primary raw materials)</td>
<td>$\overline{Y} = -33.72 + 0.25 \times f_1 + 0.42 \times f_2 + 0.88 \times f_3 - 0.00006 \times f_4$</td>
</tr>
<tr>
<td>Group 2 (auxiliary raw materials)</td>
<td>$\overline{Y} = 16.66 + 0.15 \times f_1 + 0.44 \times f_2 + 0.17 \times f_3$</td>
</tr>
<tr>
<td>Group 3 (raw materials for packing and forming)</td>
<td>$\overline{Y} = -64.10 + 0.95 \times f_1 + 0.39 \times f_2 + 0.03 \times f_3$</td>
</tr>
<tr>
<td>Group 4 (physical facilities)</td>
<td>$\overline{Y} = -145.93 + 1.49 \times f_1 + 0.22 \times f_2 + 0.80 \times f_3$</td>
</tr>
</tbody>
</table>

All the models found are adequate, as evidenced by the numerical value of the determination coefficient $R^2$, the values of the Fisher’s F-statistics with the significance level $p$ are given in the table 6.

Table 6 – Values of regression parameters by groups of stocks

<table>
<thead>
<tr>
<th>Group of material stocks</th>
<th>Determination coefficient</th>
<th>Fisher’s F-statistics</th>
<th>Level of significance (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1 (primary raw materials)</td>
<td>0</td>
<td>47</td>
<td>0</td>
</tr>
<tr>
<td>Group 2 (auxiliary raw materials)</td>
<td>0</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>Group 3 (raw materials for packing and forming)</td>
<td>0</td>
<td>329</td>
<td>0.00</td>
</tr>
<tr>
<td>Group 4 (physical facilities)</td>
<td>0</td>
<td>47</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The values of the regression parameters for factors $f_1, f_2, f_3$ are positive, which indicates a direct relationship between the market price for production stocks and the factors chosen for the example.

In the economic-mathematical model of fair value for the first group of production stocks, the value of the determination coefficient $R^2$ is 0.99, that is, the variability of the fair value for raw materials is 99% due to the influence of variability of factors $f_1$ – excess of the market price over the sales price, $f_2$ – opportunities of turnover of stocks or their costs, $f_3$ – the cost price of the product, $f_4$ – total revenue). The numerical values of the parameters of the regression equation have an important economic interpretation. So, the value of 0.25 for variable $f_1$ means that if the market retail price is higher than wholesale by 1%, then the fair value of the stock of raw materials will increase by 0.25 units. Similarly, if the cost price is increased by 1 unit, the market price of raw materials will increase by 0.88 units.

For the second group of production reserves, the value of the determination coefficient $R^2$ is 0.92, that is, the variability of the fair value of the auxiliary raw materials is 92% due to the variability of the factors, which in this case are limited to the factors $f_1$, $f_2$, $f_3$. The numerical values of the parameters of the regression equation have an important economic interpretation. So, the value of 0.95 for variable $f_1$ means that if the market retail price is higher than wholesale by 1%, then the fair value of raw materials for packing and forming will increase by 0.95 units. Similarly, with an increase in prime cost by 1 unit, the fair value of raw materials will increase by 0.03 units.

The value of the coefficient of determination $R^2$ for the fourth group is 0.99, that is, the variability of the fair value of the reserves by 99% is explained by the influence of the variability of the market price, the turnover of stocks and the cost price. A value of 1.49 for variable $f_1$ means that if the market retail price is higher than wholesale by 1%, the cost of physical facilities will increase by 1.49 units.

5. Conclusions

The urgency and insufficient knowledge of the problems of accounting for the interaction of the main flowing processes in industrial enterprises determined the need for scientific developments in optimizing the logistics function. In this paper, some problems of low adequacy of models are considered. The result of the study was the modified model, in which the optimal size of demand was determined during the formation of stocks. To solve the problem of managing production reserves, a methodical approach was suggested that makes it possible to determine in a timely manner the normative level of industrial reserves via: Model 1 (M1) – realizing the principle of calculating the initial (historical) cost of production reserves; Model 2 (M2) – providing an estimate of production stocks as of the reporting date; Model 3 (M3) – designed to establish the fair value of inventories. The
A comparative analysis of the results in simulation showed the similarity of the correlation-regression function with a high level of reliability with respect to various types of reserves.

**Bibliographic references**


### Annexes

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**Table 1**
Data for the simulation model of regression by stocks of Group 1 (primary raw materials)

<table>
<thead>
<tr>
<th>Observations</th>
<th>Market price, units</th>
<th>Influence factors</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>f1</td>
<td>f2</td>
</tr>
<tr>
<td>Enterprise 1</td>
<td>10</td>
<td>103.3</td>
<td>24.0</td>
</tr>
<tr>
<td>Enterprise 2</td>
<td>8</td>
<td>100.6</td>
<td>18.0</td>
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<tr>
<td>Enterprise 3</td>
<td>9.5</td>
<td>102.9</td>
<td>22.0</td>
</tr>
<tr>
<td>Enterprise 4</td>
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<td>23.0</td>
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<td>&lt;...&gt;</td>
<td>&lt;...&gt;</td>
<td>&lt;...&gt;</td>
</tr>
<tr>
<td>Enterprise 98</td>
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<td>100.8</td>
<td>19.0</td>
</tr>
<tr>
<td>Enterprise 99</td>
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<td>21.0</td>
</tr>
<tr>
<td>Enterprise 100</td>
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<td>102.2</td>
<td>20.0</td>
</tr>
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</table>

Table 2
Data for the simulation model of regression by stocks of Group 2 (auxiliary raw materials)

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<th>Influence factors</th>
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</tr>
<tr>
<td>Enterprise 1</td>
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<td>21</td>
</tr>
<tr>
<td>Enterprise 100</td>
<td>50</td>
<td>102.2</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 3
Data for the simulation model of regression by stocks of Group 3 (raw materials for packing and forming)
### Table 4
Data for the simulation model of regression by stocks of Group 4 (physical facilities)

<table>
<thead>
<tr>
<th>Observations</th>
<th>Market price, units</th>
<th>Influence factors</th>
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</thead>
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<tr>
<td>Enterprise 1</td>
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<tr>
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<td>102.2</td>
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</table>

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