Basics of Investment Projects
Selection for the Implementation of Regional Investment Programs in the Sphere of Social House Building

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ABSTRACT:
The problem of public housing for low income citizens is very urgent in entire world. This article is dedicated to the topical question of optimal investment projects selection for the implementation within the framework of investment programs in the sphere of public housing construction. We have formulated basic principles of formation of the optimal portfolio of projects for the investment programs for public housing construction. As selection criteria, instead of profitability and risk offered in classical portfolio theory we propose the use of social satisfaction surveys results and amount of funding to determine effectiveness of the programs. We have developed mathematical models for selection of investment projects. The two classes of models for random and determined characteristics of investment-building projects can be used by the public housing programs. These models are solved through linear programming and generic optimization. Application of them to investment projects selection within the framework of investment programs of public housing construction will allow the most effective use of funds directed to

RESUMEN:
El problema de la vivienda pública para los ciudadanos de bajos ingresos es muy urgente en todo el mundo. Este artículo está dedicado a la cuestión actual de la selección óptima de proyectos de inversión para la implementación en el marco de programas de inversión en el ámbito de la construcción de viviendas públicas. Hemos formulado los principios básicos de la formación del portafolio óptimo de proyectos para los programas de inversión para la construcción de viviendas públicas. Como criterios de selección, en lugar de rentabilidad y riesgo ofrecidos en la teoría clásica de cartera, proponemos el uso de resultados de encuestas de satisfacción social y la cantidad de fondos para determinar la efectividad de los programas. Hemos desarrollado modelos matemáticos para la selección de proyectos de inversión. Las dos clases de modelos de características aleatorias y determinadas de los proyectos de construcción de inversiones pueden ser utilizadas por los programas de vivienda pública. Estos modelos se resuelven mediante programación lineal y optimización genérica. La aplicación de los
1. Introduction

Housebuilding occupies leading position in economic and social researches. Regardless of the form of government, cozy house was always on the top of human needs hierarchy [8]. Historically, the role of the social housing sector was to provide universal access to adequate housing [13]. The major part of people nowadays buys and gets housing on market conditions; however, social housing was and is the product of interaction between state and vulnerable citizens [20]. The improvement of interaction structure of all participants of the investment process of social house building took place during the mankind evolutionary development. This sphere differs by the complexity and versatility; therefore it requires development of scientific approaches to its investigation [1]. Without competent control of investments it's impossible to implement the programs of housing sphere modernization, to form favorable investment climate in the region, to carry out structure reforms and responsible housing policy [7]. This principle works also in opposite direction – increase of people's quality of life is impossible without economic growth, therefore during the development of the programs of social house building one shouldn't forget about its economic efficiency.

One of the most important parts in the social investment programs is to choose the most efficient projects with limited resources. Modern investment theory is based with Harry Markowitz, in his work «Choice of portfolio» there were offered first mathematical models of formation of optimal securities portfolio and shown methods of their construction [15]. We have modified the task of of investment projects selection from the point of view of risk and profitability to the social satisfaction under limited financing need. Additional models for the selection of optimal investment projects of social house building create new investment qualities and perspectives.

Investment program of social house building can be constructed using different approaches depending on the choice of criteria and constrains. For the selection of optimal projects within the framework of socially-affordable housing building, we suggest the new showing SSi of social satisfaction from i-th investment project Pr. This showing can be interpreted as direct, for example, amount of families having improved housing conditions, as well as indirect, for example, decrease of tension in society, formation of new working places and so on. Social satisfaction will be evaluated subjectively by the each expert based on hypotheses and formulations offered by the authors.

Based on the two key factors we have developed two classes of models for random and determined social significance under limited financing needs of investment projects, which can be solved typical optimization programs and Boolean linear programming method. The suggested models can be effectively applied not only during the formation of social house building programs but also for any socially oriented investment programs.

2. Theoretical frameworks

During the realization of investment programs of social house building, the question of investment projects selection, implemented within the framework of the program, is put in the forefront [10]. What selection criteria should be applied? What project should be given
Financial theory became a science in 20-30 of XX century: in works of I. Fisher income discounting technique was used for the first time. By the economist's opinion, to correctly assess future income in present, one should use relation of services cost and capital cost.

Evaluation of the main (capital) assets became central topic of works of D.B. Williams, one of the most famous investors of USA. Economist has offered an approach to assets evaluation allowing controlling investment portfolio.

However, all pre-war works were created with a hypothesis about the definiteness of conditions of financial solutions, therefore, in financial analysis, elementary financial mathematics was used. But already in 1921 the work of F.H. Knight «Risk, Uncertainty and Profit» contained the first quality analysis of probability-theoretic mathematical methods of risk events influence [11].

The first investment theories appeared in the developed Western countries; economic agents have already tried to determine formula of optimum complex of investment projects for realization of investment program. This was the time when the term «investment portfolio» appeared.

The concepts of investment program and investment portfolio they are very close: many scientists don't see their principle differences. For example, Vilenskiy P., Smolyak S., Livshits V. in their scientific works draw attention to that investment portfolio is some set of projects which investor considers and then makes decision about implementation of some of them [12; 13].

In addition, the recommended set of projects is called investment program. Some scientists think that these two concepts are equal. Some economists complete the concept of investment program, opening it as an element of the investment strategy of the state. For example, Tyrtyshov J.P. pays attention that the program is a commitment that can make and mobilize resources for its implementation [14].

By our opinion, term «investment program» is wider then notion «investment portfolio» more prevalent today, but they frequently coincide in practice, therefore, within the framework of current research, these two terms will be considered as identical.

Modern investment theory is closely connected with the name of Harry Markowitz: in his work «Choice of portfolio» there was offered first mathematical model of formation of optimal securities portfolio and shown methods of their construction.

Markowitz H. models allowed translating task of investment projects selection from the point of view of risk and profitability to mathematical language. The main results of scientists-economists, received in XX century are focused on the works of Markowitz, the essence of which is that project portfolio is created on the basis of relation risk-profitability or some combinations of risk and profitability [15; 16].

To maximize profit and minimize risks, investors deal not with one asset but allocate capital to different projects. Combination of projects creates new investment qualities and perspectives. Evaluation of separate assets and their portfolios, above all, should take into account 2 showings: relation of expected risk and assets profitability. Modern investment theory takes into account both these characteristics: risk and profitability are assessed quantitatively, that allows investor to form optimal investment portfolio. Theory of «Efficient set of portfolios» remains summit of classic economic theory [17].

3. Data description and methods
It's possible to describe this theory with mathematical language in the following way: investor with a fixed sum of money should form optimal package of projects which:

1. Provide maximum profitability under the expected level of risk.
2. Provide minimum risk under the expected profitability.

Assets profitability is a change of investor's welfare in the end of analyzed period to his welfare in the beginning of the period:

\[ r = \frac{K_f - K_0}{K_0} \]  

where \( r \) — asset profitability;

\( K_f, K_0 \) — capital of investor in the end and beginning of analyzed period.

Let's indicate share or part of \( i \)-th asset in portfolio as \( X \). Structure of portfolio is described by vector \( X = \{X_1, X_2, \ldots, X_n\} \). Then, portfolio return consisting of \( n \) assets is determined:

\[ r = \sum_{i=1}^{n} X_i \tau_i \]  

where \( \tau_i \) — is the return of \( i \)-th asset.

The mean of portfolio with \( n \) projects is given by formula:

\[ P = \sum_{i=1}^{n} X_i P_i \]  

where \( P_i = \mu_i \) — mathematical expectation of return of \( i \)-asset.

If the returns of the asset are considered the independent random values, variance of the assets portfolio \( (D) \) will be determined as following:

\[ D = \sum_{i=1}^{n} X_i^2 D_i \]  

Portfolio variance is calculated by multiplying the squared weight of each security by its corresponding variance and adding two times the weighted average weight multiplied by the covariance of all individual security pairs. Thus the variance of portfolio with the dependent assets returns (which is rarely met in practice) is determined as:

\[ D = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j D_{ij} \]  

where \( D \) — variance of portfolio with dependent returns of the assets;

\( D_{ij} \) — covariance of the dependent returns of the assets \( i \) and \( j \).

In this case variance of portfolio can be calculated by the following formula:
4. Results

Based on the above-mentioned it seems necessary to develop additional models for the selection of optimal investment projects of social house building.

Assume that in the solution of the portfolio formation task the method and order of steps are determined explicitly. Hence, we can form a model working under the determined characteristics of investment projects \((P_i)\). Consider \(n\) projects:

\[
P_i = P_{i1}, P_{i2}, P_{i3}, ..., P_{in}
\]

The program of social house building \((SP)\) is derived from the suggested investment-building projects. This program can be described as some subset of investment projects from the set:

\[
SP_b \subseteq P_b
\]

Assume the investment program with characteristic vector \(X = (X_1, X_2, ..., X_n)\), where \(X_i\) takes the value 1, if the \(i\)-th project \(P_i\) is included into the program and 0 otherwise. Vector \(X\) defines the structure of the program.

For the selection of optimal projects within the framework of socially-affordable housing building, authors suggest the new showing \(SS\), of social satisfaction from \(i\)-th investment project \(P_i\).

This showing can be interpreted as direct, for example, amount of citizens having improved housing conditions, as well as indirect, for example, decrease of tension in society, formation of new working places and so on. Social satisfaction will be evaluated subjectively by the each expert based on hypotheses and formulations offered by the authors.

All social investment programs balance the contradictory conditions of limited financing and social effect requirements. Coefficient of social satisfaction, \(SS\) is determined by experts by estimating the local social satisfaction factors (fig.1).

\[
D = \mu\left(\sum_{i=1}^{n} X_i \cdot \left(\sum_{j=1}^{n} Y_j - P_j\right)\right) \approx
\]

\[
D = \text{covariance of returns of two assets. Index } \mu \text{ means } j\text{-th asset in the portfolio.}
\]

Covariance of returns of two assets in case of \(i \neq j\) is a variance. At the same time normalizing condition should be fulfilled as the components of the portfolio vector represent the fraction of the total wealth which are invested in the corresponding securities, i.e.:

\[
\sum_{i=1}^{n} X_i = 1
\]

In the classical mean-variance approach the optimal portfolio of \(n\) assets is found as following:

1. We need to find all non-negative vectors \(X = (X_1, X_2, ..., X_n)\), satisfying to the normalizing condition (7) and at the same time:

\[
P = \max \sum_{i=1}^{n} X_i \cdot P_i
\]

\[
\sum_{i=1}^{n} X_i \cdot D_{ij} \leq D_{ij}
\]

where \(P\) - expected return of the portfolio of \(n\) assets;

\(D\) - upper bound for the variance of portfolio which consists of dependent returns of assets.

2. We need to find all non-negative values \(X_i = (X_1, X_2, ..., X_n)\), satisfying to above-mentioned condition of normalization and at the same time:

\[
D = \min \sum_{i=1}^{n} X_i \cdot P_i
\]

\[
\sum_{i=1}^{n} X_i \cdot D_{ij} \geq D_{ij}
\]

where \(D\) - lower bound of the expected portfolio return.

It means that under all portfolios \(X\) we consider only those which satisfy the constrains, in particular those which yield at least an expected return \(P\). Then, among those portfolios determine the one with the smallest return variance.

3. We need to find all non-negative vectors \(X = (X_1, X_2, ..., X_n)\), satisfying to above-mentioned condition of normalizing and at the same time:

\[
P = \max \sum_{i=1}^{n} X_i \cdot P_i
\]

\[
\sum_{i=1}^{n} X_i \cdot D_{ij} \leq D_{ij}
\]

4. We need to find non-negative values \(X = (X_1, X_2, ..., X_n)\), satisfying the normalizing condition (7) and at the same time to maximize utility function:

\[
U = \max U(P, D)
\]

where \(U(P, D)\) - utility function which reflects investor's system of preferences for portfolio creation. For example, for linear utility function we can use the following formula:

\[
U = P - \frac{A}{2} D^2
\]

where \(A\) - coefficient of investor's risk sensitivity. The greater this coefficient the more sensitive to risk is investor (he acts more carefully).

There exist more formulations of the task on how to create the optimal investment portfolio. For instance, you can remove the requirement of non-negativity for variables \(X = (X_1, X_2, ..., X_n)\) by assuming the existence of risk-free asset for which \(D_{ij}=0\).

To form efficient investment portfolio we can add more tasks: for example, to suggest presence of risk-free assets, to accept zero return or offer indistinct return that seldom occurs on practice and also cancel requirement of non-negativity of portfolio projects \(X = (X_1, X_2, ..., X_n)\). Markowitz model is actively used during formation of commercial investment portfolios in different modifications. Different nonstandard solutions are developed and widely published [18]. However, with total transparency and availability of classical investment theory, it can't be used for social projects and programs. After all, as it has already been mentioned, investment programs for social house building are not developed based on the returns of the invested assets. Therefore there is a need of modification of the existing theory of optimal portfolio modeling.

\[\text{Figure 1}\]

Criteria for determing the social satisfaction of the investment projects
4.1. Model of program formation based on maximization of social significance under limited financing needs.

Criteria for determining the social satisfaction of the investment project of social housing:

- Number of families improved their living conditions
- Features of social adaptation of citizens
- Prospects for the infrastructure development of the region
- The load on the existing infrastructure in the area
- Possibilities for further construction

Showings of social effect and, consequently, social satisfaction are subjective, and every expert calculates them on his own. Showings of social satisfaction can be considered as deterministic variable or as random variable defined on the following probability space:

\[ < S, B, P > \]  \hspace{1cm} (20)

where \( S \) — sample space of all possible outcomes;
\( B \) — Borel algebra of events on the sample space \( S \), collection of all events we consider;
\( P \) — probability measure defined for every event \( A \in B \); \( P(S) = 1 \). \( A \) is coefficient of investor’s risk sensitivity.

Random variable \( SS \) of social satisfaction from every evaluable investment project, is represented by the sample:

\[ \tau = (\tau_1, \tau_2, \ldots, \tau_k) \]  \hspace{1cm} (21)

where \( k \) — sample size of expert values.

Values \( r_k \) \((k \geq 2)\) are derived by experts from the set of projects, characterized by distribution function \( F(t) \). Consider \( F_0 \) as a class of all possible distribution functions \( F(t) \), from which random sample \( r \) (return of \( i \)-th asset) can be drawn.

One more criterion of investment project selection is amount of financing of investment program \( (V) \) which can be calculated by the following formula:

\[ V = V_1 + V_2 + V_{pr} + V_g + V_{prf} + V_{nprf} + V_{other} \]  \hspace{1cm} (22)

where:
- \( V_1 \) — investment fund from government and private companies;
- \( V_2 \) — amount of profit from economic activity of facility manager (within the framework of previous programs);
- \( V_{pr} \) — amount of profit from economic activity of facility manager (within the framework of previous programs);
- \( V_g \) — amount of budget means involved into noncommercial partnership (federal, regional, municipal);
- \( V_{nprf} \) — contributions of members of noncommercial partnership;
- \( V_{other} \) — grants received by noncommercial partnership from international organizations and foreign sponsors;
- \( V_{other} \) — other attracted funds;
- \( V_2 \) — investment funds from government and private companies.

Based on the above-mentioned criteria, let’s build mathematical model of investment project for the program of social house building.

Let’s assume that characteristics and conditions of investment projects are defined (determined) that occurs rather seldom in practice; every investment project is characterized by the following showings:

- \( SS \) — coefficient of social satisfaction of the project.
- \( V \) — required amount of financial means calculated for the project.

Presented models enable to determine optimal social projects with maximal social effect under budget constrain.

Let \( V \) be suggested financing of the program during certain period. It is supposed that the total demand in financing should not exceed possible financing during the fixed period. Then the social satisfaction of members from the program consisting of \( n \) investment projects will be determined as:

\[ SS = \sum_{i=1}^{n} SS_i X_i \]  \hspace{1cm} (23)

Investment program of social house building can be constructed using different approaches depending on the choice of criteria and constrains. Based on the two key factors we propose two main models of how social house building program can be formed:
2. Model of program formation based on minimization of financing under the required level of social satisfaction

In this case, from the set of investment projects we should consider only those which form the programs which satisfy constrains, in particular those with the level of satisfaction at least as lower bound $S_{S0}$.

Then, among those programs determine the one with minimal expected financing, i.e., to find such set of vectors $X = (X_1, X_2, ..., X_n)$, where every $X$ takes one of the values $0$ or $1$ and:

$$\begin{cases}
\sum_{i=1}^{n} X_i S_{S_i} \rightarrow \max \\
\sum_{i=1}^{n} X_i V_i \leq V_0
\end{cases}$$

(24)

Value $S_{S0}$ represents the required level of social satisfaction of project members. The solution of the program formation tasks can be obtained within the help of typical optimization programs.

Mathematical expectation of return in classic theory of investment is an expected return and variance of return represents the expected risks of investors or just risks. In the suggested models, we call mathematical expectation of significance of projects and programs as expected social satisfaction and variance of significance as risk of investment program or just a risk [19].

In practice investment projects are usually characterized not by determined demand in financing but by random subjective value of social satisfaction since it depends to a large extent on experts' opinion.

Then the value of showing can be represented by the sample of expert values ($k \geq 2$), from some general population characterized by distribution function $F(t)$. Then $F$ is a class of possible distribution functions $F(t)$, from which sampling can be derived:

$$\tau_i = (\tau_{1i}, \tau_{2i}, ..., \tau_{ki})$$

(26)

Evaluation of mathematical expectation and dispersion of characteristics as applied to social satisfaction is described by the following way:

$$m_i = \frac{1}{k} \sum_{k=1}^{k} \tau_{ik}$$

(27)

$$D_i = \frac{1}{k} \sum_{k=1}^{k} (m_i - \tau_{ik})^2$$

(28)

$$\sigma_i = \sqrt{D_i}$$

(29)

where $m_i$ is mathematical expectation of social satisfaction of $i$-th investment project;

$D_i$ – variance of social satisfaction of members of $i$-th investment project;

$\sigma_i$ – standard deviation of the showing of social satisfaction of the members of $i$-th investment project.

Then mathematical expectation of social satisfaction of investment program will be determined by the formula:

$$R = \sum_{i=1}^{n} X_i \tau_i$$

(30)

For the case of dependent showings of satisfaction of investment projects, variance of socially oriented investment program will be determined as:

$$\sigma = \sum_{i=1}^{n} \sum_{j=1}^{n} X_i X_j D_{ij}$$

(31)

Where $D_{ij}$ – covariance between returns of two investment projects. In case of $i = j$ is a variance.

Assume $V_0$ is the possible amount of financing of socially oriented program during certain period. Consequently, the total demand on financing of socially oriented investment program cannot exceed the possible amount of financing.

Socially oriented investment programs can be formed and implemented during one day as well as few periods, depending on established criteria. Under random characteristics of social satisfaction the authors provide the following models of formation of optimal socially oriented projects.

4.2. Model of program formation based on the maximization of expected social satisfaction of investment program under acceptable risk and financing limitations
4.3. Model of program formation based on the minimization of financing of the program under acceptable risk limitation and achievement of required level of social significance

Such conditions can be described by the following economic and mathematical model. From the given set of investment projects, it's necessary to choose such a subset of projects that form programs which fulfill the constraints. It means that the chosen programs do not exceed the maximally acceptable risk $D_0$ and in the same time their expected social satisfaction should be at least as high as minimally acceptable level $SS_0$. Those programs form the feasible set of the programs. And our task is to choose the one program from the feasible set with the minimal financing demands.

That is, the task is to find such a set of vectors $x = (x_1, x_2, ..., x_n)$ to achieve for which constrains of achievement at least minimal expected significance of the program $SS_0$ and not to exceed maximally acceptable risk $D_0$ are mutually fulfilled. From the set of received programs choose the one with the minimal financing requirements. In mathematical language this task can be described in the following way:

\[
\begin{align*}
\sum_{i=1}^{n} x_i & \leq D_0 \quad (32) \\
\sum_{i=1}^{n} x_i & \leq \sum_{i=1}^{n} y_i \quad (33) \\
\end{align*}
\]

where $D_0$ - upper bound of the investors risks; $SS_0$ - minimal social significance, lower bound of the social significance to be reached; $D_0$ - upper bound of the investors risks, this level of risks cannot be exceeded.

Provided models will help to form optimal portfolio of investment projects in the framework of limited budget and limited acceptable risks with maximization of social satisfaction, and consequently, to form optimal package of projects with the minimal financing requirements which provides at least required amount of citizens with housing and do not exceed acceptable risks. The key task for realization of the social house building programs is an increase of region quality of life from the realization of the programs. After all, the more citizens will be provided with comfortable housing the sooner relations between society and state will stabilize. The most «vulnerable» citizens will be protected that will automatically reduce social tension in society [20].

5. Conclusion

The housing market becomes increasingly important in the overall structure of the market sector. Social comfortable housing also becomes more important due to both social and economic problems: the state of the housing stock primarily determines the quality of life of the population. Social housing also may be profitable investment from social point of view for private companies. It can enhance the prestige of investor and the loyalty of citizens to his companies; draw up new opportunities and form new social environment. Realization of investment programs of social house building is a complicated, multistage process, one of key stages of which is a selection of the most efficient projects with the framework of the program. This need is especially clearly observed in bid cities where the problem of lack of social housing is really obvious. As a result of all theoretical searches the authors have suggest showings and mathematical models of investment projects selection which will help to expert group in making a decision about the selection of the most profitable objects of social house building from the social point of view. We have developed two classes of models for random and determined characteristics of investment and building projects, which can be solved by Boolean linear programming and typical optimization programs. The suggested models can be effectively applied not only during the formation of social house building programs but also for any socially oriented investment programs.
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