REVISTA ESPACIOS

Hybrid Method for Multi-Criteria Risk Minimization

Método híbrido para la minimización de multi-criterios de riesgo

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ABSTRACT:

This paper addresses the theoretical provisions and proposes methodical approaches offering practical recommendations on finding an exact solution to minimizing risks in the course of taking business decisions in production, selling or buying products under the conditions of uncertainty and conflicting criteria. The scientific-methodical substantiation of the hybrid method of multi-criteria evaluation has been developed. The proposed conceptual model is based on the principles of system analysis. The authors present methodological and practical recommendations using a specific example of risk assessment and minimization. The proposed methods and tools allow evaluating any possible risks at all stages of searching for solutions in production, manufacturing or buying various products in order to avoid adverse economic consequences in business activities.
Keywords: models, risks, minimization, hybrid models, multi-criteria methods.

RESUMEN:

Este documento aborda las disposiciones teóricas y propone enfoques metódicos que ofrecen recomendaciones prácticas para encontrar una solución exacta para minimizar los riesgos en la toma de decisiones comerciales en la producción, venta o compra de productos en condiciones de incertidumbre o conflictivas. Se ha desarrollado la justificación del método híbrido de evaluación multicriterio. El modelo conceptual propuesto se basa en los principios del análisis del sistema. Los autores presentan recomendaciones metodológicas y prácticas utilizando un ejemplo específico de evaluación y minimización de riesgos. Los métodos y herramientas propuestos permiten evaluar los posibles riesgos en todas las etapas de búsqueda de soluciones en la producción, fabricación o compra de varios productos para evitar consecuencias económicas adversas en las actividades comerciales.
Palabras clave: modelos, riesgos, minimización, modelos híbridos, métodos de criterios múltiples

1. Introduction

The relevance of the presented research is stipulated by the need to find an optimal solution in business activities that meets many different criteria often contradicting each other. The solution to the problem is reached through enumerating possibilities to find

solutions, each of which surpasses the other by at least one of the specified criteria (Sukhorukova I.V. & Chistyakova N.A., 2018). A large number of such solutions are called the best ones according to Pareto.

The task of searching for a set of optimal solutions is quite time-consuming, with classical methods of optimization deemed practically unsuitable. To solve this problem it is necessary to find not only a set of solutions that meet the specified criteria, but also to achieve the maximum distinction between these solutions, and accordingly, to find a larger number of these options (Belton V., 2002; Hesham K.A. & Salih O.D, 2008). Therefore, it is necessary to find a point in the area of acceptable solutions to the issue under consideration, which minimizes or maximizes the whole set of characteristicz. Thus, for example, a seemingly simple task of generating a product range includes a lot of important issues: selection of model and brand of goods, logistics chain, time of purchase and delivery of goods, choosing a financial transaction scheme, solution tool, choosing an expert to advise on minimizing the risks of each operation, implementing the plan as a whole and avoiding fraud to successfully reach the objective. At the same time, there is a clear desire to achieve maximum efficiency at the lowest cost, choosing an investment solution when it is necessary to get the maximum benefit with the least risk. (Chistyakova N.A. & Sukhorukova I.V, 2018). However, the decision which simultaneously would satisfy the set of inconsistent requirements, as a rule, does not exist, and hence the decision is accepted in the conditions of risk. In this case, it is necessary to consult with specialists to ensure the implementation of the solution and particularly, to use non-formalized methods of searching for a solution. Finding an optimal solution provides a balanced solution to financial and economic problems, significantly saves money, reduces costs and reduces waste production, making it possible to obtain high-quality products.

2. Literature survey

At present, the problem of building particular models and a set of methods of risk assessment and minimization exists in all spheres of human activity. The uncertainty and multi-criteria nature of the areas under consideration, especially the diversity of characteristics stemming from the differences in the assessment and dynamics of the studied objects, create a significant difficulty and obstacles in conducting such a research. Therefore, to link them into a successful model composition is the most difficult task. To date, no fundamental research has been carried out in relation to risk areas, although certain research developments have taken place (Tikhomirov N.P. & Tikhomirova T.M., 2016; Fomin G.P. 2018; DEMIN Sergey S. et al., 2018). Shown that optimal decision making is based on the use of various non-formalized and formalized methods and the degree of their hybridization is determined by different circumstances. It was also indicated that in order to improve the quality of the solution, it is necessary to apply multi-criteria optimization, especially in a situation when the initially selected criteria do not give us full confidence that our final decision is optimal and correct. In that regard, it was pointed out that with approximate estimates of some values obtained by formalized methods it is easier to make a final decision.

Initially, the problem of multi-criteria optimization was raised by the Italian economist V. Pareto in 1904 in the mathematical study of commodity exchange in the trade process (Censor Y., 1977). To cater for this process the sequential concessions method, the ideal point method and the convolution method were suitable. For the purpose of the present study, it is more convenient to use the method based on the convolution of criteria, indicators or characteristics of the object, where instead of a set of specific criteria one scalar criterion obtained by a combination of specific criteria was considered. Among the latter there are multiplicative, additive and mixed methods of convolution of criteria. It is assumed that the criteria are comparable and normalized and their weighted coefficients, which characterize the importance of each criterion among the set, are defined. After that, a new objective function is built and the task of optimization of the scalar criterion is solved (Lootsma F.A. , 1999; Shi-Woei & Lin Ming-Tsang Lu, 2012 ; Kwiesielewicz M., 2004; Gawlik R., 2017). Thus, for example, there is a method of analytic hierarchy

process (AHP), first proposed by the American researcher Thomas Saaty (Saaty, T. L. 1980). This method is aimed not only at helping to arrive at the best single decision, but also at ranking possible decisions and criteria used in decision making. Building a decision hierarchy makes possible to carry out comparative assessment of the contributions of alternative solutions and to find the one that best describes a certain ranking criterion and satisfies the conditions of the task (Saaty T. L. 1994; Saaty, T. L. 2012; Koyun Yılmaz S. & Ozkir V. , 2018). The authors have taken advantage of these methods and combined them into a model composition, integrating them with other mathematical methods used in the process of taking business decisions in manufacturing, selling or buying products under the conditions of uncertainty and conflicting criteria.

3. Methodology

It is important to note that the success of commercial activities of any organization is determined by the appropriate formation of the manufactured and sold product range, which is generated on the basis of objective market research, analysis and forecasting demand. In this regard, an important task of the company is to form such a range of products or to prepare an appropriate pre-shipment order, so that they can ensure the optimal allocation of available monetary funds for production or purchase of products with minimal risks.

The subject matter of the research is the mechanisms of formation of risk assessment methods for objects of different nature of business activity in production, selling or buying products under the conditions of uncertainty and conflicting criteria.

Methods of the research include both non-formalized and formalized methods of modeling, principles of system analysis, including structuring and quantification of risk study issues while creating a combined version of multi-criteria methods of optimization. The authors also used statistical methods of ranking the characteristics of the object of evaluation, method of paired comparisons, methods of integral programming, Hungarian method, methods of reduction, method of trial and error.

The purpose of this study is the scientific substantiation and creation of a hybrid method of multi-criteria assessment and minimization of risks in business activities under the conditions of uncertainty. There are economic and mathematical methods which allow to quantitatively justify the choice of solution in conditions of multi-criteria. However, the researchers are interested in a comprehensive method that would consider the opinions of business participants and the characteristics of the object itself, a method that could objectively, justifiably and preferably quickly and simply minimize risks. To this end, we decided to utilize the existing arsenal of both non-formalized and mathematical methods, to integrate their advantages, minimizing their disadvantages and to offer a mixture in the form of a hybrid method of multi-criteria risk minimization. The task of multi-criteria optimization is to search for a vector of objective variables satisfying the given restrictions and optimizing the vector function, the elements of which correspond to the objective functions. Therefore, the task of multi- criteria optimization is reduced to the task of optimization with one scalar objective function. These functions form mathematical description of the criterion of satisfaction and, as a rule, they conflict with each other. That is why it is necessary to find such a solution, at which the value of the objective functions would be acceptable for the problem setter.

4. Results

This paper proposes methodical approaches and offers practical recommendations on finding an exact solution for a businessman how to ensure the forthcoming demand for a new product and minimize risks at decision-making under the conditions of uncertainty. The authors developed the scientific and methodical substantiation and designed a conceptual economic model of multi-criteria evaluation and risk minimization in the process of product acquisition. Let us consider how a trade organization sets and solves the task by employing in the course of the decision-making a set of necessary methods

for assessing risks on buying / selling new products with the minimum risk under the conditions of uncertain demand. Deviation of revenue from sales in comparison with the expected planned amount determines the risk. In the presented example, it is necessary to make an optimal allocation of monetary funds for the purchase of coffee machines with minimal risk in the formation of product range based on the characteristics presented in Table 1.

Table 1
Comparative characteristics of coffee machines

№	Technical and Economic Characteristics	Coffee machines Models					
		Melitta	Siemens	Saeco	Philips	Bosch	Nespresso
1	Average price (thousand rubles)	30	31	32	28	27	29
2	Coffee Beans / Ground Coffee	C.B.	C.B./Gr.	C.B./Gr.	C.B.	C.B./Gr.	Capsules
3	Capacity (l)	1,2	2,4	1,8	1,8	1,7	0,9
4	Power (W)	1400	1700	1850	1850	1600	1400
5	AutoPower Off	+	−	+	−	+	+
6	Height (cm)	33	46	33	33	39	26
7	Weight (kg)	8,3	8	7,2	7	6,6	4,5

First, let us find a solution using analytic hierarchy process (AHP). The method of hierarchy analysis includes decomposition using hierarchies and synthesis by finding relationships through experts' judgments and linear convolution (Saati T.L., 2008; Wang Y. & Luo Y., 2009; Barron F.H., 1992). First, we build a hierarchical structure of the task: goal, criteria, sub-criteria, alternatives. Then, we apply the method of estimation of the strength of judgments (Table 2).

Table 2
Nine-point scale of assessment of the power of judgement

Importance Degree	Definition	Explanation
1	Equivalent importance	Two actions bring in equal contribution to the goal attainment
3	Certain predominance of the importance of one action (factor index) over another, weak dependence	Experience and judgement give some preference to one action over another
5	Significant or great importance	Experience and judgement give strong preference to one action over another

7	Greater importance or evident importance				The preference of one action over the other is very strong, its superiority is almost obvious			
9	Absolute importance				Evidence in favor of one action over another is to the highest degree convincing			
2, 4, 6, 8	Intermediate values between adjacent scale values				A situation where a compromise solution is needed			
Inverse values of these numbers	If the i action when compared to j is assigned one of the above numbers, then the action j is assigned an inverse value in comparison with i				If there is an integer number above the diagonal, then under the diagonal shall be its opposite value			
Rational value	Ratio that appears on a given scale				To obtain a consistent matrix, n numerical values are required			

Let us determine the priority vector for the characteristics using the method of analytic hierarchy process (AHP) in relation to the upper level of the hierarchy. For this purpose, we build a matrix of paired comparisons of coffee machine characteristics (Table 3) and calculate the normalized priority vector for it.

Table 3
Matrix of paired comparisons of coffee machine characteristics

	Coffee Beans/Ground Coffee	Capacity	Power	Auto PowerOff	Height	Weight	vi	wi
Coffee Beans/Ground Coffee	1	2	3	4	6	5	21.00	0.343
Capacity	1/2	1	2	3	5	4	15.50	0.253
Power	1/3	1/2	1	2	4	3	10.83	0.177
AutoPowerOff	1/4	1/3	1/2	1	3	2	7.08	0.116
Height	1/6	1/5	1/4	1/3	1	1/2	2.45	0.040
Weight	1/5	1/4	1/3	1/2	2	1	4.28	0.070

Using this table, we form the square symmetrically reciprocal matrix A:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 6 & 5 \\ \dfrac{1}{2} & 1 & 2 & 3 & 5 & 6 \\ \dfrac{1}{3} & \dfrac{1}{2} & 1 & 2 & 4 & 3 \\ \dfrac{1}{4} & \dfrac{1}{3} & \dfrac{1}{2} & 1 & 3 & 2 \\ \dfrac{1}{6} & \dfrac{1}{5} & \dfrac{1}{4} & \dfrac{1}{3} & 1 & \dfrac{1}{2} \\ \dfrac{1}{5} & \dfrac{1}{4} & \dfrac{1}{3} & \dfrac{1}{2} & 2 & 1 \end{pmatrix}$$

We determine approximately the coordinates v_i of the unnormalized principle eigenvector $\vec{v}$ as follows: the i-th coordinate v_i of $\vec{v}$ is the sum of the elements of the i-th row of the matrix A. In the example above, We determine approximately the coordinates v_i of the unnormalized principle eigenvector $\vec{v}$ as follows: the i-th coordinate v_i of $\vec{v}$ is the sum of the elements of the i-th row of the matrix A. In the example above, $\vec{v} = \left(21, \frac{31}{2}, \frac{65}{6}, \frac{85}{12}, \frac{49}{20}, \frac{257}{60}\right)$. These calculations are presented in Table 3. Next, we form the normalized priority vector $\vec{w}$ relative to the upper level of the hierarchy. For this purpose, we normalize $\vec{v}$ by dividing each coordinate of $\vec{v}$ by S, where $S = v_1 + v_2 + \cdots + v_n$. Then, in the example above, $S = \frac{3669}{60} = 61.15$ and consequently

$$\vec{w} = \left(^1/_S\right) \cdot \vec{v} = \frac{1}{3669}(1260,\ 930,\ 650,\ 425,\ 147,\ 257).$$

The matrix A is called a judgment matrix or called a pairwise comparison matrix, entries of which represent the relative importance ratios of the alternative Pi over the alternative Pj (where i, j=1,2..., n) with respect to a common criterion. Ideally, such a matrix should satisfy a property $a_{ij} = a_{ik}a_{kj}$ (because if we say we prefer Pi to Pk a_{ik} times and Pk to Pj a_{kj} times, then then it seems quite reasonable to assume that we should prefer Pi to Pj $a_{ik}a_{kj}$ times. Recall that a positive square matrix of order n is called to be transitive (or consistent) if $a_{ij} = a_{ik}a_{kj}$ for all $i, j = 1, \ldots, n$. However, experts may compare alternatives inconsistently and, as a result, obtain a non-transitive matrix of pairwise comparisons. The question is how much inconsistency is acceptable to continue the analytic hierarchy process. In AHP, a measure of the inconsistency is the so-called consistency ratio. In order to avoid the inconsistency of expert judgments in relation to each other, it is necessary to calculate the consistency ratio for our matrix.

Let us determine the degree of consistency of the judgment matrix. It is known that any positive matrix A has a positive real eigenvalue λ_{max}, such that $|\lambda| < \lambda$max for all other complex eigenvalues λ of A. This maximal real eigenvalue λ_{max} is the largest in absolute value, see (Horn, R.A. & Johnson, C.R., 2013) Theorem 8.4.4. The positive eigenvector, corresponding to the maximal eigenvalue λ_{max}, is called the principle eigenvector.

Let us find approximately maximal eigenvalue to estimate the consistency of judgments: $\lambda_{max} \approx \frac{\sum_{i=1}^{n} u_i}{S}$ where $\vec{u} = A\vec{v}$ and u_i are coordinates of $\vec{u}, i = 1, \ldots, n$. In our example, $\sum_{i=1}^{n} u_i = 17512/45$ and therefore $\lambda_{max} \approx 6.364. \approx 6.364$. We use this value in the process of assessing alternatives, calculating the consistency index or consistency ratio.

The consistency ratio is denoted by CR and defined as the fraction CR = CI/RI comparing the consistency index (CI) of the judgment matrix in question versus the consistency index of a random-like matrix (RI).

The consistency index, CI is as follows: $CI = \frac{\lambda_{max} - n}{n-1}$; $CI = \frac{6.364-6}{6-1} = 0.0728..$ Finally, we find the consistency ratio (the proportion of inconsistency) CR = CI/RI, (Saaty, 1994; J. A. Alonso & M. T. Lamata, 2006), where RI is given in the following table

Table 4
Consistency indices for a randomly generated matrix.

n	1	2	3	4	5	6	7	8	9	10	11

RI	0	0	0,58	0,9	1,12	1,24	1,32	1,41	1,45	1,49	1,51

The value of a consistency ratio (CR) of 0.10 or less is used as valid value to continue the AHP analysis (Saaty T. L., 1994; Saaty T. L., 2012). If the consistency ratio is greater than 0.10 (CR > 0.1), then this indicates a significant violation of the judgment logic made by the expert when filling the matrix. So, in the case of CR > 0.1, in order to improve consistence the expert is invited to revise his judgment used to construct the judgment matrix. To provide us with an improved judgment matrix with a better consistency, it is necessary to reconsider the data on the properties and characteristics of the products being compared.

In our example above, where n=6, we get CR = 0.0728/1.24= 0.0587. The pair comparison matrix is reasonably consistent because CR <0.1.

Similarly, we calculate the priorities of compared objects in relation to the criteria. Calculation of priority vectors is carried out in the direction from the upper levels to the lower levels, with account for links between elements belonging to different levels.

Table 5
Criterion: Coffee Beans/Ground Coffee.

	Me	Si	Sa	Ph	Bo	Ne	W1
Me	1	1/3	1/3	1	1/3	3	0,107
Si	3	1	1	3	1	5	0,249
Sa	3	1	1	3	1	5	0,249
Ph	1	1/3	1/3	1	1/3	3	0,107
Bo	3	1	1	3	1	5	0,249
Ne	1/3	1/5	1/5	1/3	1/5	1	0,0403

Let us check the consistency of judgments in the matrix. We have $\lambda max = 6.2$ and the size of comparison matrix is 6, thus the consistency index is CI= (6.2-6)/(6-1)= 0.04. Then we have CR=0.04/1.24=0.0323 <0.1. Thus, the comparison matrix is reasonably consistent.

Table 6
Criterion: Coffee Machine Capacity

	Me	Si	Sa	Ph	Bo	Ne	W2
Me	1	1/4	1/2	1/2	1/2	1	0.0707
Si	4	1	2	2	3	5	0.32
Sa	2	1/2	1	1	2	5	0.217
Ph	2	1/2	1	1	2	4	0.198
Bo	2	1/3	1/2	1/2	1	3	0.138

	Ne	1	1/5	1/5	1/4	1/3	1	0.0562

Let us check the consistency of judgments in the matrix. Since λmax = 6.194, CI= (6.194-6)/6 = 0,0388 and CR = 0.0388/1.24 = 0.0313. The comparison matrix is reasonably consistent because 0.0313 < 0.1.

Table 7
Criterion: Coffee Machine Power

	Me	Si	Sa	Ph	Bo	Ne	W3
Me	1	1/5	1/7	1/7	1/4	1	0.0351
Si	5	1	1/3	1/3	2	5	0.175
Sa	7	3	1	1	5	7	0.308
Ph	7	3	1	1	5	7	0.308
Bo	4	1/2	1/5	1/5	1	5	0.14
Ne	1	1/5	1/7	1/7	1/5	1	0.0344

Let us check the consistency of judgments in the matrix. We obtain λmax=6.774, CI= (6.774-6)/5 =).1555 and CR=0.155/1.24=0.125. The result is negative. Since the consistency ratio is greater than 0.10, this comparison matrix is not reasonably consistent and it is necessary to revise the judgments to locate the cause of the inconsistency and correct it.

Table 8
Criterion: Coffee Machine Power

	Me	Si	Sa	Ph	Bo	Ne	W4
Me	1	5	1	5	1	1	0.227
Si	1/5	1	1/5	1	1/5	1/5	0.0455
Sa	1	5	1	5	1	1	0.227
Ph	1/5	1	1/5	1	1/5	1/5	0.0455
Bo	1	5	1	5	1	1	0.227
Ne	1	5	1	5	1	1	0.227

It turns out that λmax= 6. Hence, CI = 0 and CR=0/1.24=0. In this case we have an ideal judgment matrix, i.e. the matrix is completely consistent.

Table 9
Criterion: Coffee Machine Height.

	Me	Si	Sa	Ph	Bo	Ne	W5
Me	1	4	1	1	2	1/2	0.184

	Me	Si	Sa	Ph	Bo	Ne	
Si	¼	1	1/4	1/4	1/3	1/5	0.044
Sa	1	4	1	1	2	1/2	0.184
Ph	1	4	1	1	2	1/2	0.184
Bo	½	3	1/2	1/2	1	1/3	0.113
Ne	2	5	2	2	3	1	0.291

Let us check the consistency of judgments in the matrix: λmax=6.153, CI=(6.153-6)/5 = 0.036 and CR=0,0306/1.24 =< 0.1. Therefore, the comparison matrix is reasonably consistent.

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Table 10
Criterion: Weight.

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	Me	Si	Sa	Ph	Bo	Ne	W6
Me	1	1	1/2	1/3	1/5	1/7	0.0515
Si	1	1	1	1/2	1/3	1/5	0.0654
Sa	2	1	1	1	1/3	1/5	0.0897
Ph	3	2	1	1	½	1/5	0.125
Bo	5	3	3	2	1	1/4	0.231
Ne	7	5	5	5	4	1	0.438

We have λmax=6.433 and, consequently,CI = (6.433-6)/5 = 0.0866; CR=0.0866/1.24=0.0698. Since this value of 0.0698 for the consistency CR is less than 0.10, the comparison matrix is reasonably consistent.

We carry out a hierarchical synthesis. We consistently define the vectors of priorities of alternatives concerning the elements of the hierarchy at all hierarchical levels.

$$\|\vec{W_1}, \vec{W_2}, \vec{W_3}, \vec{W_4}, \vec{W_5}, \vec{W_6}\| \cdot \vec{w} =$$

$$\begin{Vmatrix} 0.107 & 0.0707 & 0.0351 & 0.227 & 0.184 & 0.0515 \\ 0.249 & 0.32 & 0.175 & 0.0445 & 0.0442 & 0.0654 \\ 0.249 & 0.217 & 0.308 & 0.227 & 0.184 & 0.0897 \\ 0.107 & 0.198 & 0.308 & 0.0445 & 0.184 & 0.125 \\ 0.249 & 0.138 & 0.14 & 0.227 & 0.113 & 0.231 \\ 0.0403 & 0.0562 & 0.0344 & 0.227 & 0.291 & 0.438 \end{Vmatrix} \begin{Vmatrix} 0.343 \\ 0.253 \\ 0.177 \\ 0.116 \\ 0.0401 \\ 0.07 \end{Vmatrix}$$

$$= \begin{Vmatrix} 0.0981162 \\ 0.2089704 \\ 0.2348134 \\ 0.1627174 \\ 0.1921343 \\ 0.1027913 \end{Vmatrix}.$$

As a result, generalized weight coefficients of each coffee machine model were determined. Let us rank them in descending order of their generalized priorities: 1. Saeco (0.235); 2. Siemens (0.209); 3. Bosch (0.192); 4. Philips (0.163); 5. Nespresso

(0.103); 6. Melitta (0.098).

Now we will apply different combined methods to solve the same problem on search of the optimal decision ensuring risks minimization in business decision-making process in manufacturing, selling or buying products in the conditions of uncertainty and conflicting criteria. Comparing the characteristics of coffee machines, we build a matrix of paired comparisons (Table 11), the elements of which are determined by the following rule:

$$b_{ij} = \begin{cases} 1, & \text{if the compared characteristics are equivalent in importance;} \\ 2, & \text{if the } i^{th} \text{ characteristic is more important than the } j^{th} \text{ characteristic;} \\ 0, & \text{if the } i^{th} \text{ characteristic is less important than the } j^{th} \text{ characteristic.} \end{cases}$$

Table 11
Paired Comparison Matrix of Characteristics.

	coffee beans/ ground coffee	capacity	power	automatic twining on	height	weight	Si	Vi	Rank
coffee beans/ ground coffee	1	2	2	2	2	2	11	0,31	1
capacity	0	1	2	2	2	2	9	0,25	2
power	0	0	1	2	2	2	7	0,19	3
automatic twining on	0	0	0	1	2	2	5	0,14	4
height	0	0	0	0	1	0	1	0,03	6
weight	0	0	0	0	2	1	3	0,08	5

Upon filling in the matrix with comparison elements, we find the sum of points for each indicator in rows: $S_i = \sum_{j=1}^{n} b_{ij}$

where n is the number of indicators and n = 6.

The correctness of filling the matrix is determined by the equation:

$$\sum_{j=1}^{n} S_i = n^2 = 36$$

Then we determine the weight (the point assessment) of the V_i indicators by the formula: $V_i = S_i/n^2$.

The priority of the indicators is distributed over the rank Ri, which is proportional to the value of the weight coefficient: the higher its value, the higher the rank, and the highest value Vi corresponds to Ri = 1. For the objective calculation of the objective function values for each coffee machine, let us enter the score for each indicator. We consider that the better the qualitative content of the indicator, the higher its score. Then, using the input data, the following scoring matrix can be constructed (Table 12):

Table 12
The scoring assessment of coffee-machine characteristics

objective assessments of the quality	1	2	3	4	5
coffee beans /					coffee beans /

	ground coffee	capsules	–	coffee beans	–	ground coffee
capacity	0.9-1.2	1.2-1.5	1.5-1.8	1.8-2.1		2.1-2.4
power	1400-1500	1501-1600	1601-1700	1071-1800		1801-1900
automatic twining on	No	–	–	–		Yes
height	46-50	41-45	36-40	31-35		25-30
weight	8.5-9.4	7,8	6.5-7.4	5.5-6.4		4.5-5.4

Using the data from this table we determine the values of integral evaluations of coffee machine quality without taking account of the price by the following formula:

$$Q(i) = \sum_{i=1}^{n} f_{ij} \cdot V_j \,,$$

where $Q(i)$ is the overall quality assessment of the i-th coffee machine, f_{ij} is the objective assessment of the quality of the i-th coffee machine relative to the j-th indicator, Vj – the point assessment of the j-th indicator of Pj.

In matrix form, the calculation of general priorities is written as follows:

$$
\begin{pmatrix}
3 & 1 & 1 & 5 & 4 & 2 \\
5 & 5 & 3 & 1 & 1 & 2 \\
5 & 3 & 5 & 5 & 4 & 3 \\
5 & 3 & 5 & 1 & 4 & 3 \\
5 & 3 & 2 & 5 & 3 & 3 \\
1 & 1 & 1 & 5 & 5 & 5
\end{pmatrix}
\begin{pmatrix}
0.31 \\ 0.25 \\ 0.19 \\ 0.14 \\ 0.03 \\ 0.08
\end{pmatrix}
=
\begin{pmatrix}
2.35 \\ 3.7 \\ 4.31 \\ 3.75 \\ 3.71 \\ 2.0
\end{pmatrix}
$$

Therefore, the overall priorities of coffee machines (also called final priorities) are equal: Q (Melitta) = 2.35; Q (Siemens) = 3.7; Q (Saeco) = 4.31; Q (Philips) = 3.75; Q (Bosch) = 3.71; Q (Nespresso) = 2.

Optimal allocation of monetary funds on formation of optimal product range with the minimum risk should be carried out in quantities and shares defined after rationing operation.

Table 13
Allocation of Funds with Minimal Risk.

coffee machines models	Saeco	Siemens	Bosch	Philips	Nespresso	Melitta	Amount
Q	4,31	3,7	3,71	3,75	2	2,35	19,82
optimal distribution Qopt.	0,217	0,187	0,187	0,189	0,1	0,119	1
uniform distribution Qun.	0,167	0,167	0,167	0,167	0,167	0,167	1
price of purchase, thousand RUR	15	15	13	14	14	15	–
selling price, thousand	32	31	27	28	29	30	–

RUR							
risk loss of profit R1, percent	5%	2%	2%	2.2%	–	–	11.2%
R1 , thousand RUR	96	31	27	56	–	–	210
risk of frozen funds R1, percent	–	–	–	–	6.7%	4.8%	11.5%
R2, thousand RUR.	–	–	–	–	145	90	235

We consider that under conditions of uncertainty and unclear consumer demand, the level of comparison to determine the risk is the uniform distribution of allocated funds in the amount of 1 million rubles in equal shares $Q_{un.} = 1/6 = 0.167$. It should be noted that the price of purchase accounts for about 50% of the selling price. The ratio of integral quality characteristic, for example, for $Q(Saeco) = 4.31$ to the total sum of qualities $\sum Q(i) = 19.82$ determines the share of the allocated amount with the minimum risk for the purchase of $Q_{opt.} = 4.31/19.82 = 0.217$. Accordingly, the risk of lost profit (R1) will be determined by the difference $0.217 - 0.167 = 0.05$, i.e. R1 = 5%. Similar calculations have been performed for all brands of coffee machines and are presented in Table 13. On the basis of these calculations, the Pareto diagram can be completed. In general, the risk of loss of profit will constitute 11.2%, and the risk of frozen funds will amount to (11.5%), which is also a part of the lost risk and makes up 11.2%+11.5% = 22.7% of the allocated funds, i.e. 445 thousand rubles, which is quite essential for their further turnover. In order to minimize risks, it is necessary to distribute funds based on the Pareto diagram. Thus, the existing formalized methods of risk assessment based on the results of risk identification and the identification of risk indicators can be used in combination with mathematical methods to conduct a risk assessment, but this process requires a great deal of expertise and extensive knowledge of risk theory.

5. Conclusions

It should be noted that the weight of the indicators in the paired matrix of characteristics in the hybrid method and AHP almost coincide. Calculation of the correlation relationship between weights of criteria, which were calculated in both methods, found that there is a strong correlation between them (0.95). However, it should be noted that the solution of the problem by the AHP method and the hybrid method still gives a discrepancy, because the ranked lists of coffee machines do not coincide. This is due to the fact that the application of the nine-point scale for estimation of the power of judgments does not allow using it successfully and accurately in the presented variant, as there is no binding of numerical values of characteristics to the scale points. Therefore, it is necessary to create a special interval scale with reference to the number of product characteristics, and the scale must be observed for all characteristics. Our ability to do so depends heavily on the factors that we can measure in order to compare. The analysis of the strengths and weaknesses with the method of the analytic hierarchy process has shown that the use of a scale from 1 to 9 increases the degree of sensitivity, but is highly dependent on the expert. Calculation of the consistency index helps avoid violations in the experts' logic regarding the importance (significance) of the indicators and criteria, however, this did not work out when solving the problem by this method and other evaluation criteria are obviously needed. In general, the AHP method is complex and time-consuming to use, especially when increasing the number of characteristics, for example, up to 10. The scale is intermittent. It should be modified to adjust the interval scale to the physics, the nature of the task. To find out the significance of the criteria, an interval scale with quantitative grading of characteristics and a matrix of paired comparisons should be used, which is the easiest for the experts to perceive. Analyzing and using the advantages of different methods and eliminating disadvantages, we have

developed a hybrid method of multi-criteria risk minimization, which is easy enough to solve the problems of multi-criteria optimization, because it takes much less time to solve the problem. Thus, the existing non-formalized and formalized methods of risk assessment based on the results of risk identification and identification of risk indicators can be used in a rich combination with mathematical methods to assess and minimize risks, but this process requires expertise, extensive knowledge and involvement of specialists of different profiles.

The scientific-methodical substantiation of the hybrid method of multi-criteria evaluation was carried out and the authors offered conceptual model of minimization of risks of various products, suitable for use in various spheres of entrepreneurial activity. The model includes as a basis the principles of system analysis – structuring and quantification of the risk study issue, creation of the combined version of multi-criteria method of optimization. Development of methodical and practical recommendations has been carried out on a specific example of risk assessment and minimization. The proposed methods and tools allow for the assessment and consideration of possible risks at all stages of the search for solutions for the production, manufacture or purchasing various products to avoid adverse economic consequences in business activities.

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