Economic and Mathematical Modeling of the Process of Costing at the Car Industry Companies

Modelado económico y matemático del proceso de cálculo de costos en la industria automovilistica

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Received: 28/08/2017 • Approved: 03/10/2017

ABSTRACT:
At present, the car industry companies face the task of determining the minimum price of an item in view of all factors that influence the conversion cost. The authors offer the methodology of determining the causes of deviation from the indicators of the operative and production plan, based on the results of correlation and regression analysis, on the basis of which a program of correcting measures for eliminating the causes of deviations is developed for AVTOVAZ PJSC. The received results allow eliminating these deviations and increasing the effectiveness of organization of the operative and production planning of car industry companies.

Keywords: planning, price, economic and mathematical models, car industry companies, factors, analysis.

RESUMEN:
En la actualidad, las empresas de la industria del automóvil se enfrentan a la tarea de determinar el precio mínimo de un artículo a la vista de todos los factores que influyen en el costo de conversión. Los autores ofrecen la metodología para determinar las causas de desviación de los indicadores del plan operativo y de producción, basándose en los resultados del análisis de correlación y regresión, sobre la base de la cual se desarrolla un programa de medidas correctoras para eliminar las causas de las desviaciones. AVTOVAZ PJSC. Los resultados recibidos permiten eliminar estas desviaciones y aumentar la efectividad de la organización de la planificación operativa y de producción de las empresas de la industria del automóvil. Palabras clave: planificación, precio, modelos económicos y matemáticos, empresas de la industria del automóvil, factores, análisis.

In the conditions of the globalizing market, the car industry companies face a serious task of determining the total cost of production and to calculate the item price during compilation of the production program[4]. At the same time, the car industry companies use two main methods of calculation of cost: job cost and normative [2].

The job cost method allows for calculation of a separate production order for one or several items. The cost includes the expenses related to production of the order and a part of expenditures for servicing the production and management of the company, divided by the
order proportionally to the selected base.

A huge drawback of the job cost method consists in receiving the whole information on the results of the performed order only after its factual completion, which does not allow determining the production cost fully at the stage of planning. That’s why there are certain risks during establishment of the contractual price; for minimizing them, an approximate price is calculated before the start of production of the order, and the final price is calculated after the production of the order, which is usually higher than the approximate one [12].

The normative method of planned calculations is considered to be most progressive. Its main peculiarity is the usage for the planning of items’ cost of the scientifically substantiated and progressive norms of resources’ expenditures, which determines the necessity for the corresponding normative base at the company [13].

The viewed methods of calculation of cost are effectively used at the car industry companies, which do not have a full production cycle. In view of the peculiarities of car industry companies with the full production cycle, a system of intra-company relations is built which emerge in the process of execution of the production program of a certain assortment. These relations are based on the necessary quantitative and cost provision of production departments with raw materials, work pieces, and parts, which might be self-engineered and provided by external suppliers. In view of the peculiarities of interrelations, it is expedient to use the economic and mathematical modeling for accounting of production costs and formation of products’ cost [7, 14].

For the purpose of substantiation and systematization of formation of costs of car industry companies during manufacture of market products, in view of interrelations of production departments, two economic and mathematical models were developed:

1. Stochastic economic and mathematical model of production program planning (Figure 1);
2. Stochastic economic and mathematical model of planning of market goods’ cost.

Mathematically, these relations are expressed in the following way:

$$Z = \sum_{i=1}^{i=3} Z_i$$  \hspace{1cm} (1)

where: $Z$ – quantity of products (product program), which are to be manufactured, pcs; $i$ – stage of technological chain:

1. procuring,
2. processing,
3. assembling.

Using Formula 1, the final value of product cost is calculated, which consists of the sum of production costs, which take into account all factors that influence the value of market products’ cost.

Fig. 1
Stochastic economic and mathematical model of the production program planning (market products)
Machine building company with a full production cycle
The first model allows determining the production program for each item of the market products and describes organizational, technological, and other relations between the production departments that are connected by the one process (procuring, processing, and assembling) of manufacture of market products, according to the planned tasks. Besides, this model shows connections to the suppliers of raw materials, work pieces, and parts necessary for the main production, and the consumers of the issued products.

More than 60% in the formation of the finished products’ price depends on the suppliers – that’s why it is advised to use various approaches to coordinated interaction between the customer (assembly plant) and suppliers for reduction of cost [5].

Very often, the researchers determine this interaction by selecting the volume of the additional effect obtained by the suppliers in case of realization of the customer’s managing influences. As the additional effect, which is assigned to each element, is a part of the total effect, such approach – according to certain authors – will allow excluding ineffective variants of solving the management task and evaluating the effectiveness of managing the system on the whole.

The value of the additional effect could be obtained by selecting the special stimulating functions that are a variable part of the target function, or by changing various coordinating parameters of the models of elements’ functioning [10]. For example, some specialists allow redistributing the additional effect of the customer company between the suppliers on the basis of changing the contractual prices.

\[
C_{PROD.TOT} = \sum_{k=1}^{m} C_{1k} \cdot Z_{1k} + \sum_{k=1}^{m} C_{2k} \cdot Z_{2k} + \sum_{k=1}^{m} C_{3k} \cdot Z_{3k}
\]

where \(C_{PROD.TOT}\) – total cost of market products.

In the general form, market products' cost is the following mathematical dependence:

\[
C_{PROD.TOT} = \sum_{i=1}^{k=m} \sum_{l=3}^{i=i} C_{ik} \cdot Z_{lk}
\]

where \(i\) – the stage of technological chain of production:

1 – procuring production;
2 – processing production;
3 – assembling production.

The quantity of products manufactured after a certain stage of the technological chain is calculated with the following formula:

\[
Z_{i} = \sum_{j=1}^{j=q} \sum_{i=1}^{i=a} Z_{ij}
\]

where: \(Z\) – quantity of products that pass the technological process; \(j\) – products that pass the production process, but has different ways of bringing it to the main production:

- \(a\) – products supplied by suppliers;
- \(b\) – products of own production and sent to the consumer;
- \(c\) – products of own production for internal consumption.

In the expanded form, this mathematical model has the following form:

\[
\begin{align*}
z_{1a} & \Rightarrow z_{1} = z_{1a} + z_{1e} \Rightarrow Z_{1} = z_{1a} \\
z_{2} + z_{1e} & \Rightarrow z_{2} = z_{2b} + z_{2e} \Rightarrow Z_{2} = z_{2b} \\
z_{3a} + z_{2e} & \Rightarrow z_{3} = z_{3b} \Rightarrow Z_{3} = z_{3b}
\end{align*}
\]
This organizational and economic mechanism consists in the following: the customer appoints two prices for parts $P_{i1}$ and $P_{i2}$. When all customer’s requirements as to volumes, terms of supply, and quality of parts are met, the customer pays for the parts for a higher price $P_{i2}$. If the customer’s requirements are not met in full, the customer pays for the parts for the lower price $P_{i1}$.

Besides, another effective mechanism of redistribution of the additional effect between the supplier and the customer is awarding a bonus. If the customer’s requirements are not met, he does not pay any bonus to the suppliers. Coordination of economic interests of the supplier and the customer requires for the premium to be equal or higher than the sum of possible losses of the suppliers and lower than the additional customer’s effect [3]. However, as the practice often shows, these methods require additional financial expenditures of the customer company.

In this case, a stimulus for the quality of the supplier's work is increase of the order volume. Let us view this by the example of the two-level system “supplier-customer”, which consists of the assembly plant and the final multitude of suppliers.

The assemble plant purchases $n$ quantity of materials or parts, with each of them having $m$ alternative suppliers. The target function of the customer company is minimization of expenditures of the supply department for purchasing the material assets and expenditures for elimination of suppliers' defects, expenditures due to delayed supply, etc. [8, 9]:

$$F(x_{ij}, \beta_{ij}) = \sum_{i=1}^{n} \sum_{j=1}^{m} [p_{ij}x_{ij} + \alpha \beta_{ij}x_{ij}^2] \rightarrow \min$$

where $p_{ij}, x_{ij}$ – price and volume of purchases of $i$-th part of $j$-th supplier, accordingly; $\alpha$ – size coefficient, which converts the point indicators into expenditures; $\beta_{ij}$ – integral point evaluation of non-conformities, which characterizes the quality $i$-th part of $j$-th supplier.

The economic sense of the first part of the target function $F(x_{ij}, \beta_{ij})$ – cost of the parts purchased by the assembly plant, second part – expenditures for elimination of defects, non-conformities, warranty repair, delayed supplies, etc. The more the number of the estimate indicators, the larger the share of defects and non-conformities with the $j$-th supplier, and, accordingly, the lower the quality of $i$-th part.

The volume of the purchased part of $i$-th type with all suppliers is connected to the production program of the assembly plant with the following ratio:

$$\sum_{i=1}^{n} X_{ij} = \gamma_{i}N,$$  \hspace{1cm} (7)

where $\gamma_{i}$ – number of parts of $i$-th type in the finished product; $N$ – the company’s production program.

The cost of all purchased parts is imposed with the following financial limitations

$$\sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij}x_{ij} \leq F \hspace{1cm} (8)$$

where $F$ – financial budget of the supply department.

The mechanism of stimulating the supplier for the quality of his work consists in the following. The supply department solves the task of non-linear programming (6), (7), (8) and determines the optimal volume of supplies $x_{ij}$ for $i$-th type of parts for $j$-th supplier, based on the interests of the assembly plant. The suppliers that provide more profitable ratio of parts’ price and quality of work receive the larger volume of order and larger profit [11].

Let us use the Lagrange multiplier for solving the task of constrained extremum. Let us write down the Lagrangian:

$$L(x_{ij}, \lambda_{1i}, \lambda_{2}) = \sum_{i=1}^{n} \sum_{j=1}^{m} [p_{ij}x_{ij} + \alpha \beta_{ij}x_{ij}^2] - 
\sum_{i=1}^{n} \lambda_{1i} \left[ \sum_{j=1}^{m} x_{ij} - \gamma_{i} \cdot N \right] - \lambda_{2} \left[ \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij}x_{ij} - \Phi \right]\hspace{1cm} (9)$$

where $\lambda_{1i}, \lambda_{2}$ – Lagrange multipliers.

Let us write down the necessary conditions of optimality:
At that, the customer company determines the optimal volume of shipment for each supplier. The suppliers that provide a better ratio of price for parts and quality of work receive a larger volume of order and, therefore, larger profit.

After establishment of interrelations between the supplier and the customer, it is necessary to pay attention to products’ coming through all stages of the technological production chain.

\[
\begin{align*}
\frac{\partial L}{\partial x_{ij}} &= p_{ij} + 2\alpha \beta_{ij} x_{ij} - \lambda_{1i} - \lambda_2 p_{ij} = 0, \\
\frac{\partial L}{\partial \lambda_{1i}} &= \sum_{i=1}^{n} x_{ij} - \gamma_i N = 0, \\
\frac{\partial L}{\partial \lambda_2} &= \sum_{i=1}^{n} \sum_{j=1}^{m} [p_{ij} x_{ij} - \Phi] = 0
\end{align*}
\]

Solving the system of \((n \cdot m + n + 1)\) equations with \((n \cdot m + n + 1)\) unknowns, we receive

\[
\lambda_2 = \frac{\Phi}{r} - \frac{1}{r} \sum_{i=1}^{n} \frac{d_i}{c_i} \gamma_i N + 1
\]

\[
\lambda_{1i} = \frac{\gamma_i N}{c_i} + \frac{d_i}{r c_i} \left[ \sum_{i=1}^{n} \frac{d_i}{c_i} \gamma_i N - \Phi \right], \quad i = 1, n
\]

\[
x_{ij} = \frac{1}{2\alpha \beta_{ij}} \left[ \frac{\gamma_i N}{c_i} + s \left( \frac{d_i}{c_i} - p_{ij} \right) \right], \quad i=1, n; \; j=1, m
\]

In the formulae (11, 12, and 13), \(c_i, d_i, r, s\) are constants that are determined with the following formulae:

\[
c_i = \sum_{i=1}^{m} \left[ \frac{1}{2\alpha \beta_{ij}} \right]
\]

\[
d_i = \sum_{j=1}^{m} \left[ \frac{p_{ij}}{2\alpha \beta_{ij}} \right]
\]

\[
s = \frac{1}{r} \left[ \sum_{i=1}^{n} \frac{d_i}{c_i} \gamma_i N - \Phi \right]
\]

\[
r = \sum \left[ e_i - \frac{d_i^2}{c_i} \right]
\]

where \(e_i\) – constant.

Expression (10) determines the volume of order of \(i\)-th part \(y\) of \(j\)-th supplier depending on the integral evaluation of non-conformities \(\beta_{ij}\), price \(p_{ij}\), volume of financial assets \(\Phi\), and production program \(N\).

Improvement of quality of \(i\)-th part \(y\) of \(j\)-th supplier, which is expressed in reduction of the integral evaluation of non-conformities \(\beta_{ij}\), increases the volume of order for this supplier. At that, the volume of order depends on two components: the volume of the production program of the assembly plant \(N\) and price of \(i\)-th part of \(j\)-th supplier \(p_{ij}\).

The second component is preceded by the coefficient \(s\), which increases or decreases the influence of the price. Decrease of price for \(i\)-th part of \(j\)-th supplier \(p_{ij}\) leads to increase of the volume of order \(x_{ij}\).

At that, the customer company determines the optimal volume of shipment for each supplier. The suppliers that provide a better ratio of price for parts and quality of work receive a larger volume of order and, therefore, larger profit.

After establishment of interrelations between the supplier and the customer, it is necessary to pay attention to products’ coming through all stages of the technological production chain.
The second economic and mathematical model (Figure 2) describes the principles of cost formation at the stage of planning, is based on qualitative interrelations of the first model and ensures formation of cost of each stage of the technological process and cost of finished market products. In this concept, it describes the process of interrelations between the participants of intra-company relations in view of internal planned and estimate prices and allows considering the following terms and limitations:

1. interaction between the subjects of intra-company relations that participate in formation of market products’ cost;
2. prices for market products after each stage of the production process, which could be expressed by the following mathematical dependence, presented by Formula 19.

\[
\sum_{k=1}^{m} U_{1(MAT)k} + \sum_{k=1}^{m} U_{1k} \Rightarrow \sum_{k=1}^{m} C_{1k} \cdot Z_{1k} + \sum_{k=1}^{m} II_{1k} \Rightarrow U_{1(TOB)k}
\]

\[
\sum_{k=1}^{m} U_{2(3MAT)k} + \sum_{k=1}^{m} U_{2k} \Rightarrow \sum_{k=1}^{m} C_{2k} \cdot Z_{2k} + \sum_{k=1}^{m} II_{2k} \Rightarrow U_{2(TOB)k}
\]

\[
\sum_{k=1}^{m} U_{3(KOMILT)k} + \sum_{k=1}^{m} U_{3k} \Rightarrow \sum_{k=1}^{m} C_{3k} \cdot Z_{3k} + \sum_{k=1}^{m} II_{3k} \Rightarrow U_{3(TOB)k}
\]

where \( U \) – price for \( k \)-th item; \( M \) – expenditures for production of \( k \)-th item; \( C \) – cost of products of \( k \)-th item; \( Z \) – quantity of products of \( k \)-th item; \( II \) – planned (received) profit from \( k \)-th item; \( k \) – name of item; \( m \) – quantity of articles.

Fig. 2

Stochastic economic and mathematical model of planning of market products’ cost
The developed model of planning of full cost of finished products allows considering the whole totality of initial data necessary for consideration of the production costs at the planning stage. At that, the following parameters should serve as the initial data:

- the volume of products manufacture in the natural and monetary terms;
- prices, tariffs, and taxes rates;
- norms of expenses of human and material labor per product item;
- norms of amortization expenses for full recovery of the key assets;
- norms of allowances for social and medical insurance, etc.;
- plans of technical development, improvement of organization of production, labor, and management;
- materials of analysis of economic activity for the previous period [1, 2].

Based on the offered economic and mathematical models, the norms for preparing the schedules of work of all production departments involved in the technological process of the
finished products are prepared, and the parameters to be met during realization of the plan are
determined.

Application of correlation and regression analysis during realization of the analysis function
allows determining system deviations that emerge in organization of operative & production
planning and offering the measures for their elimination. The tasks of this function are as follows:

- analysis of the production possibilities for executing the plan;
- determination of the reasons due to which the plan of issue of work pieces, products’ parts,
  and finished products cannot be performed;
- analysis of the new reasons for the purpose of their elimination.

This analysis allows for determination of the current deviations in the system of operative and
production planning and for prevention of their possible appearance by means of application of
correlations and regression analysis. The algorithm of such analysis is given in Figure 3; it has
eight blocks [6].

During execution of the operative and production plan, certain conditions may arise that do not
allow fulfilling it in full – they should be taken into account in the process of planning of the
course of production. These conditions could be eliminated during determining the factors that
influence the non-fulfillment of the operative and production plan (Fig. 3, block 1).

The multitude and diversity of the factors that influence the non-fulfillment of the operative and
production plan causes the necessity for their ranking and evaluation: determining the level of
influence and selecting 1-3 most significant factors (Fig. 3, block 2); for that, correlation
analysis (correlation dependence) is used, which allows determining the connection between
the factors and the determined deviation. This provides a right of choice of 1-3 most significant
factors.

**Fig. 3**
The algorithm of determining the reasons for deviation from
the indicators of the operative and production plan
Start

Determining the factors that influence the non-fulfillment of operative and production plans

Evaluation and selection of the factors that influence the non-fulfillment of operative and production plans

Determining the reasons that influence the factor

Determining the influence of the reasons on the factor

Is the influence determined?

Yes

Evaluation of the level of influence of the reason on the factor

Selection of the reasons that perform the maximum influence on the factor

Development of the program of correcting actions

No

Exclude the reason

Are the deviations liquidated?

Yes

Finish
After selecting the factors that perform significant influence on the deviation, the reasons that influence them are determined (Fig. 3, block 3), the quantity and diversity of which depend on the character of the factors. Determination of causal connections is performed with the use of calculation of the correlation coefficient. The received results of the correlation analysis dictate the following sequence of actions.

If the influence between the reason and the factor is not determined, i.e., the connection is weak or absent, this reasons is excluded from the list (Fig. 3, Block 5), as it does not influence the factor. In this case, we’re coming back to block 3 (Fig. 3) for additional, detailed determination of the reasons that influence the factor.

If the results are positive, i.e., the connection between the reasons and the factor is determined, we pass to evaluation of the level of the reason’s influence on the factors with application of regression analysis (Fig. 3 block 6), which, on the basis of statistical multi-factor experiment, allows creating a regression equation. Its variables show the direction and level of influence of a separate factor or their combination on the studied object.

After all reasons of the chain are ranked as to the level of their influence, the reasons that perform the maximum influence on the factor are determined (Fig. 3, block 7). This allows determining the common reasons that influence all factors and determining the tools and means of their elimination or minimization of the reasons’ influence on the factor. For that, a program of correcting actions is developed (Fig. 3 block 8), which allows considering deviations at the stage of the planning of the production course and bringing down their influence on the final result to the minimum.

As a result of analysis of block 8, two options for actions arise. If the deviations are eliminated as a result of analysis, the events offered in the program of correcting actions are implemented; otherwise (deviations are not eliminated as a result of analysis), it is necessary to get back to block 3 (Fig. 3) and to determine the reasons that influence the factors.

Determination of the factors that influence non-fulfillment of the operative and production plan in the main aspect of this algorithm.

The evaluating parameter should be unit cost of market products, as it takes into account the costs of production and expenses that arise in case of non-fulfillment of the plan. The cost is influenced by a range of factors the sum of which, according to formula (20), determines the factual (optimal) full cost.

\[
C_{FACT, FULL} = C_{OPT} = f_1 + f_2 + f_3 + f_4 + f_5 + f_6 + ... + f_k = \sum_{i=1}^{k} f_i,
\]

where \( i = 1, 2, 3, ..., k \) – number of factors that influence the value of full cost; \( f_1 \) – general expenditures that are a part of the cost; \( f_2 \) – losses from defects; \( f_3 \) – warranty service; \( f_4 \) – losses from delays; \( f_5 \) – shortfall of material goods at the storage; \( f_6 \) – payments for employees.

Evaluation and selection of the factors that influence the non-fulfillment of the operative and production plan, i.e., unit cost of market products, could be performed with the correlation analysis, which allows solving the following research tasks: measuring the strength of connection; selecting the factors that largely influence the resulting attribute; determining the unknown reasons for connection; creating the correlation model and evaluating its parameters; verifying the significance of connection’s parameters; interval value of parameters.

The methodology of determining the reasons for deviation from the indicators of the operative and production plan, based on the correlation and regression analysis, is shown in Table 1.

<table>
<thead>
<tr>
<th>Block No.</th>
<th>Block name</th>
<th>Block description</th>
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<tbody>
<tr>
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<td>Methodology of determining the reasons for deviation from the indicators of the operative and production plan, based on the correlation and regression analysis</td>
</tr>
<tr>
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</tbody>
</table>
| **Block1** | Determination of the factors that influence the deviation from the operative and production plan | All factors (external and internal) are determined, which influence the deviation from the operative and production plan. 

\[
\text{DEV} = f_1 + f_2 + f_3 + f_4 + f_5 + f_6 + \ldots + f_k = \sum_{i=1}^{k} f_i, \quad (21)
\]

where \( f \) – factors that influence the non-fulfillment of the operative and production plan; \( i (i=1, 2, 3, \ldots, k) \) – number of the factors that influence the non-fulfillment of the operative and production plan. |
| **Block2** | Evaluation and selection of the factors that perform significant influence on deviation from the operative and production plan | Study of the connection between variables is performed on the basis of two-dimensional correlation model, in which the measure of connection is the paired correlation coefficient or its modification – selected correlation coefficient \( r \), which is calculated on the basis of selected data as a certain empirical measure of connection. The selected paired correlation coefficient of dependence is calculated with the following formula: 

\[
r = \frac{S(xy)}{\sqrt{S(xx)S(yy)}}, \quad (22)
\]

where 

\[
S(xx) = \sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - \frac{1}{n} \left( \sum_{i=1}^{n} x_i \right)^2
\]

\[
S(yy) = \sum_{i=1}^{n} (y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{1}{n} \left( \sum_{i=1}^{n} y_i \right)^2
\]

\[
S(xy) = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i y_i - \frac{1}{n} \left( \sum_{i=1}^{n} x_i \right) \left( \sum_{i=1}^{n} y_i \right)
\]

where \( x, y \) – evaluated variables; \( n \) – number of pairs of data. 

Correlation coefficient takes values in the range \(-1 \leq r \leq 1\). The presence of connection could be determined on the basis of the level of connection between the variables. 

In practice, for the purpose of convenience of calculations in the two-dimensional correlation analysis a correlation table and correlation fields, which have a form of diagram with the values of two attributes, are built. Each point of the diagram has its coordinates in \( x \) and \( y \). 

As a result of calculation, the factors with the correlation coefficient in the interval \( 0.75 < |r| < 1.0 \) are selected – i.e., the ones with strong and fundamental connection. |
| **Block3** | Determination of the factors that influence the factor | All reasons that influence the factor are determined. |
| **Block4** | Establishing the influence of the reason on the factor | The connection between the variables is studied on the basis of two-dimensional correlation model, in which the measure of the connection is the paired correlation coefficient or its modification – selected correlation coefficient \( r \), which is calculated on the basis of selected data as a certain empirical measure of connection. Based on the results of calculation, the decision on establishment or lack of connection between the reason and the factor is made. |
| **Block5** | Exclusion of the reason from the list | The reasons that do not have medium, strong, or fundamental connection in the process of execution of block 5 are excluded. This is caused by the fact that these reasons are not connected to the factors, or the connection is weak, so it is not expedient to use them for elimination of deviations. |
| **Block6** | Evaluation of the level of the reason’s influence on the factor | This block is fulfilled on the basis of statistical multi-factor experiment, which can be performed with the use of regression analysis and regression equation. 

The regression equation is selected according to the sense of the studied phenomenon similarly to the traditional methods of economic statistics, the only different being the coefficients' of regression equation (growth curves) not selected with the help of the packages of statistical processing but calculated according to the special algorithms of mathematical statistics. 

In practice, the following types of regression equations are used: 

- \( \bar{y} = \beta_0 + \beta_1 x \) – two-dimensional linear; 
- \( \bar{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k \) – multi-dimensional linear; 
- \( \bar{y} = \beta_0 + \beta_1 x + \beta_2 x^2 + \ldots + \beta_k x^k \) – multinomial; |
Realization of the offered methodology ensures its sustainable position in the conditions of
tough competition in the market and allows planning the rational use of the existing resources.
It is based on correlation and regression analysis, which allows for determining the reasons for
deviation from the indicators of the operative and production plan of issue of market products
at the stage of planning the course of production.

Application of this methodology was approbated by the example of the metallurgical plant
AVTOVAZ PJSC, namely the work of the SPO filling lime, which is used for manufacture of the
block of cylinders. This item was analyzed, as the SPO filling line is the only one at the plant
and it ensures full production of the block of cylinders casting. At that, in case of deviations in
the work of this line, there appears a threat to non-execution of the production plan of market
products of the whole plant. The calculations that are performed with this methodology are
performed for perspective, for the purpose of elimination of emerging deviations from the set
operative and production planning norms of fulfilling the plan of issue of market products. After
determination of the problems in organization of the operative & production planning and
determination of the area that requires detailed analysis, this allowed developing the measures
for correction actions.

\[
\hat{y} = \beta_0 + \beta_1 \frac{1}{x} - \text{hyperbolic},
\]
\[
\hat{y} = \beta_0 x_1^{\beta_1} x_2^{\beta_2} \cdots x_k^{\beta_k} - \text{powerlike}.
\]
As a result of regression analysis, we receive dimensional (numerical) variables, which show the
level of influence of the reason on the factor.

<table>
<thead>
<tr>
<th>Block 7</th>
<th>Selection of the reasons that perform the maximum influence on the factor</th>
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<tbody>
<tr>
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<td>The reasons could be selected with the method of comparison of dimensional variables, which have the higher value and pose the largest interest for further research.</td>
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<th>Блок 8</th>
<th>Development of the program of the correcting actions</th>
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<td>The measures that allow eliminating the reasons determined in the course of the regression analysis are developed.</td>
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<tr>
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<td>If the measures allowed for elimination of the determined problem, the plan of the company’s work is revised, and the deviations are eliminated.</td>
</tr>
<tr>
<td></td>
<td>If the offered measures did not allow for elimination of deviations, it is necessary to perform the expanded analysis of reasons that influence the factors, i.e., return to block 4.</td>
</tr>
</tbody>
</table>
The performed calculation for determination of the correlation dependence between each factor \((f_1, f_2, ..., f_6)\) and the cost of casting of a cylinders block is shown in Table 2.

For the convenience of calculations in the two-dimensional correlation analysis, a correlation table and correlation fields, which are a diagram with total values of two attributes, are built. Each point of this diagram has its coordinates \(x\) and \(y\) (Fig. 4).

Using the package of analysis MS Excel and the initial data, it is possible to calculate the value of the correlation coefficient for each pair. The results of calculation are given in Table 3.

### Table 2 – Results of calculations of the paired correlation coefficient

<table>
<thead>
<tr>
<th>Pair No.</th>
<th>(S(xy))</th>
<th>(S(xx))</th>
<th>(S(yy))</th>
<th>(r)</th>
<th>(r^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^{st}) pair</td>
<td>101,852.659</td>
<td>60,044.7904</td>
<td>195,733.0401</td>
<td>0.9396</td>
<td>0.8827</td>
</tr>
<tr>
<td>2(^{nd}) pair</td>
<td>2,318.7572</td>
<td>44.261</td>
<td>195,733.0401</td>
<td>0.7878</td>
<td>0.6206</td>
</tr>
<tr>
<td>3(^{rd}) pair</td>
<td>52,372.1144</td>
<td>20,080.3411</td>
<td>195,733.0401</td>
<td>0.8354</td>
<td>0.8942</td>
</tr>
<tr>
<td>4(^{th}) pair</td>
<td>24,576.321</td>
<td>4,420.7741</td>
<td>195,733.0401</td>
<td>0.8355</td>
<td>0.6979</td>
</tr>
<tr>
<td>5(^{th}) pair</td>
<td>14,574.7575</td>
<td>1,213.6395</td>
<td>195,733.0401</td>
<td>0.9457</td>
<td>0.7002</td>
</tr>
</tbody>
</table>

The performed calculations show the following:

- 1\(^{st}\) and 3\(^{rd}\) pairs have strong connection \((0.75 < r^2 < 0.9)\);
- 2\(^{nd}\), 4\(^{th}\), and 5\(^{th}\) pairs have weak connection \((0.5 < r^2 < 0.75)\).

That’s why the connections between full cost of cylinder block casting and total expenses and losses from the fault time of equipment are interesting, as they directly influence the cost of products – namely, the full cost of cylinder block casting.

Then it is necessary to determine the reasons that influence the factors. The viewed factors are influenced by a lot of reasons that are presented in Table 2.

### Table 3 – Selected data of market products for cylinder block casting

<table>
<thead>
<tr>
<th>The viewed components (factors)</th>
<th>Selected data for the year, RUB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>January</td>
</tr>
<tr>
<td>(f_1) – total expenses that are a part of unit cost of cylinder block casting (resources and materials, parts, semi-processed materials and services of the production character, fuel and energy, social allowances, equipment, tools, and packaging)</td>
<td>488.1</td>
</tr>
<tr>
<td>(f_2) – losses from defects (losses from technological defects)</td>
<td>65.7</td>
</tr>
<tr>
<td>(f_3) – warranty service</td>
<td>Not available, as this cost is formed at the beginning of the technological chain</td>
</tr>
<tr>
<td>(f_4) – losses from down time (expenses for equipment maintenance)</td>
<td>325.2</td>
</tr>
<tr>
<td>(f_5) – shortfall of material values at the storage (shop’s expenses)</td>
<td>149.1</td>
</tr>
<tr>
<td>(f_6) – payments for employees (wages)</td>
<td>81.6</td>
</tr>
</tbody>
</table>

**Fig. 4**

Equations of the paired linear correlation for various factors of full cost of cylinder block casting
<table>
<thead>
<tr>
<th>Factor</th>
<th>Reasons that influence the factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$ – general expenditures that a part of unit cost of products</td>
<td>The state of equipment (down time of equipment due to various reasons, time of repairs, capabilities of equipment, its wear, etc.). Number of hired workers (their qualification and possibilities). Used technologies. Rationalization offers.</td>
</tr>
<tr>
<td>$f_2$ – losses from defects</td>
<td>The state of equipment (down time of equipment due to various reasons, time of repairs, capabilities of equipment, its wear, etc.). Qualification of workers. Production plan. Commissioning work.</td>
</tr>
<tr>
<td>$f_3$ – warranty services</td>
<td>The state of equipment (down time of equipment due to various reasons, time of repairs, capabilities of equipment, its wear, etc.). Terms of the agreement.</td>
</tr>
<tr>
<td>$f_5$ – lack of material assets at</td>
<td>Theft.</td>
</tr>
</tbody>
</table>
The table shows that the factors have similar reasons which influence their value. That’s why application of the correlation coefficient helps to determine the connection between the reasons and the factors, and the further calculations take into account the reasons that are present in more than three factors. These are the state of equipment, qualification of workers, and the number of hired workers.

Calculation of the paired linear correlation coefficient during determination of connection between the factor of general expenses that are a part of full cost and its reasons is shown in Table 5 and Figure 4.

Calculation of the paired linear correlation coefficient during determination of connection between the factor of loss from down time of equipment and its reasons is shown in Table 6 and Figure 5.

Table 5
Calculation of the paired linear correlation coefficient during determination of connection between the factor of general expenses that are a part of full cost and its reasons

<table>
<thead>
<tr>
<th>Period</th>
<th>General expenses that are a part of full cost</th>
<th>State of equipment</th>
<th>Number of hired workers</th>
<th>Used technologies</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>488.0409</td>
<td>129.9383</td>
<td>128.1156</td>
<td>117.9628</td>
</tr>
<tr>
<td>February</td>
<td>468.9942</td>
<td>128.0511</td>
<td>122.7254</td>
<td>117.9628</td>
</tr>
<tr>
<td>March</td>
<td>481.4392</td>
<td>132.1348</td>
<td>136.2413</td>
<td>117.9628</td>
</tr>
<tr>
<td>April</td>
<td>474.5824</td>
<td>127.5276</td>
<td>124.3029</td>
<td>117.9628</td>
</tr>
<tr>
<td>May</td>
<td>502.1469</td>
<td>137.6742</td>
<td>122.1076</td>
<td>117.9628</td>
</tr>
<tr>
<td>June</td>
<td>503.8088</td>
<td>131.6465</td>
<td>132.5779</td>
<td>117.9628</td>
</tr>
<tr>
<td>July</td>
<td>523.4041</td>
<td>138.0128</td>
<td>128.1234</td>
<td>117.9628</td>
</tr>
<tr>
<td>August</td>
<td>529.8634</td>
<td>145.9147</td>
<td>129.9514</td>
<td>117.9628</td>
</tr>
<tr>
<td>September</td>
<td>660.1133</td>
<td>181.9947</td>
<td>146.9181</td>
<td>117.9628</td>
</tr>
</tbody>
</table>
Table 6
Calculation of the paired linear correlation coefficient during determination of connection between the factor of losses from worn time of equipment and its reasons

<table>
<thead>
<tr>
<th>Period</th>
<th>Losses from down time of equipment</th>
<th>Operative and production planning</th>
<th>Preventive maintenance</th>
<th>Factual equipment life</th>
<th>Physical and moral wear of equipment</th>
<th>Qualification of workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>325.2617</td>
<td>56.5197</td>
<td>26.6715</td>
<td>85.75</td>
<td>50.5818</td>
<td>82.0491</td>
</tr>
<tr>
<td>February</td>
<td>325.2617</td>
<td>58.5543</td>
<td>11.0589</td>
<td>85.75</td>
<td>70.5818</td>
<td>93.0491</td>
</tr>
<tr>
<td>March</td>
<td>325.2617</td>
<td>52.0491</td>
<td>10.7337</td>
<td>85.75</td>
<td>65.5818</td>
<td>72.0491</td>
</tr>
<tr>
<td>April</td>
<td>411.9524</td>
<td>93.9252</td>
<td>51.4941</td>
<td>85.75</td>
<td>69.3937</td>
<td>115,5826</td>
</tr>
<tr>
<td>May</td>
<td>411.9524</td>
<td>107.5196</td>
<td>21.8335</td>
<td>85.75</td>
<td>89.3937</td>
<td>96,5826</td>
</tr>
<tr>
<td>June</td>
<td>411.9524</td>
<td>101.7523</td>
<td>22.2455</td>
<td>85.75</td>
<td>84.3937</td>
<td>106.5826</td>
</tr>
<tr>
<td>July</td>
<td>414.1671</td>
<td>96.381</td>
<td>16.5665</td>
<td>85.75</td>
<td>89.8743</td>
<td>117.2093</td>
</tr>
<tr>
<td>August</td>
<td>414.1671</td>
<td>93.0677</td>
<td>24.4356</td>
<td>85.75</td>
<td>85.8743</td>
<td>117.2593</td>
</tr>
<tr>
<td>September</td>
<td>414.1671</td>
<td>103.1276</td>
<td>23.6073</td>
<td>85.75</td>
<td>89.8743</td>
<td>107.2093</td>
</tr>
<tr>
<td>October</td>
<td>429.2836</td>
<td>96.1596</td>
<td>21.8935</td>
<td>85.75</td>
<td>73.1546</td>
<td>116.4873</td>
</tr>
<tr>
<td>November</td>
<td>429.2836</td>
<td>81.1346</td>
<td>24.4692</td>
<td>85.75</td>
<td>93.1546</td>
<td>120.4873</td>
</tr>
<tr>
<td>December</td>
<td>429.2836</td>
<td>109.8966</td>
<td>25.7571</td>
<td>85.75</td>
<td>87.1546</td>
<td>111.4883</td>
</tr>
<tr>
<td>Correlation coefficient – r</td>
<td>0.9050</td>
<td>0.3924</td>
<td>no</td>
<td>0.7701</td>
<td>0.8836</td>
<td></td>
</tr>
</tbody>
</table>
Using the obtained data, let us determine the value of the paired correlation coefficient, which allows determining the connection between the reason and the factor. The results of calculations are shown in Table 7.

**Table 7**

Results of calculations of the paired correlation coefficient, applied during determination of connection between reasons and the factor
### Results of regression analysis of the influence of various reasons on general expenditures which are a part of unit cost

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard error</th>
<th>t-statistics</th>
<th>Lower 95%</th>
<th>Upper 95%</th>
<th>Lower 95.0%</th>
<th>Upper 95.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y-crossing</td>
<td>-476.55</td>
<td>0</td>
<td>-476.5441</td>
<td>-476.5441</td>
<td>-476.5441</td>
<td>-476.5441</td>
</tr>
<tr>
<td>State of equipment (X1)</td>
<td>2.17</td>
<td>0</td>
<td>2.168043</td>
<td>2.168043</td>
<td>2.168043</td>
<td>2.168043</td>
</tr>
<tr>
<td>Number of employed workers (qualification) (X2)</td>
<td>1.78</td>
<td>0</td>
<td>1.783502</td>
<td>1.783502</td>
<td>1.783502</td>
<td>1.783502</td>
</tr>
<tr>
<td>Used technologies (X3)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The conducted calculations showed the following:
- 1<sup>st</sup> and 2<sup>nd</sup> pairs have strong connection ($0.75 < r^2 < 0.9$) in general expenditures, and 1<sup>st</sup> and 5<sup>th</sup> pairs - in losses due to down time of equipment;
- medium connection ($0.5 < r^2 < 0.75$) – 4<sup>th</sup> pair (in losses due to down time of equipment);
- weak connection and its absence – all other pairs.

That's why it is possible to conclude that the largest influence on the viewed factors is performed by the state of equipment, number of hired workers, and the applied system of operative and production planning.

Using the data presented in Table 7, the following reasons are excluded: used technologies; preventive maintenance; physical term of equipment life; physical and moral wear of equipment. This is caused by the fact that these reasons are not related to the factors or it is weak, so it is not expedient to use them for further analysis.

Using the initial data for the reasons and factors and the MS Excel, let us determine the following regression dependence, which is presented in Tables 8 and 9.

On the basis of the received data, let us build the regression dependence in the standardized scale: formula (26) – for regression dependence of reasons and general expenditures, which are a part of the products' cost; formula (27) – for regression dependence of reasons and losses from the down time of equipment.

### Table 8

Results of regression analysis of the influence of various reasons on general expenditures which are a part of unit cost
Results of the regression analysis of the influence of various reasons on losses due to down time of equipment

<table>
<thead>
<tr>
<th>Reason</th>
<th>Coefficients</th>
<th>Standard error</th>
<th>t-statistics</th>
<th>Lower 95%</th>
<th>Upper 95%</th>
<th>Lower 95.0%</th>
<th>Upper 95.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y-crossing</td>
<td>-482.37</td>
<td>0</td>
<td>65,535</td>
<td>-482.37</td>
<td>-482.37</td>
<td>-482.37</td>
<td>-482.37</td>
</tr>
<tr>
<td>Operative and production planning (X1)</td>
<td>1.15</td>
<td>0</td>
<td>65,535</td>
<td>1.153105</td>
<td>1.153105</td>
<td>1.153105</td>
<td>1.153105</td>
</tr>
<tr>
<td>Preventive maintenance (X2)</td>
<td>0.28</td>
<td>0</td>
<td>65,535</td>
<td>0.27764</td>
<td>0.27764</td>
<td>0.27764</td>
<td>0.27764</td>
</tr>
<tr>
<td>Factual term of equipment life (X3)</td>
<td>0</td>
<td>0</td>
<td>65,535</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Physical and moral wear of equipment</td>
<td>0.52</td>
<td>0</td>
<td>65,535</td>
<td>0.521467</td>
<td>0.521467</td>
<td>0.521467</td>
<td>0.521467</td>
</tr>
<tr>
<td>(X4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Qualification of workers (X5)</td>
<td>0.97</td>
<td>0</td>
<td>65,535</td>
<td>0.967173</td>
<td>0.967173</td>
<td>0.967173</td>
<td>0.967173</td>
</tr>
</tbody>
</table>
\[ T = 2.17x_1 + 1.78x_2 + 0x_3 \]  
\[ T = 1.15x_1 + 0.28x_2 + 0x_3 + 0.52x_4 + 0.97x_5 \]

The sign and value of the regression coefficient show the direction and the level of influence on the studied object of a separate factor or combination of factors. As the signs are positive in the viewed example, it means that the above parameters perform growing influence on the evaluated reasons, i.e., lead to their increase.

Taking into account the values of the regression equations of formula (26) and (27), it is possible to conclude that for the general expenditures which are a part of the products’ unit cost the significant values are \( b_1 = 2.17 \) – the state of equipment and \( b_2 = 1.78 \) – number of employed workers (qualification of workers), and for losses due to down time of equipment \( b_4 = 1.15 \) – the applied system of operative and production planning. Other parameters can be neglected. Accordingly, there are violations in the use of equipment, caused by deviations due to their neglect in the process of planning of the production course, which are accumulated and lead to failures in the work of the system of operative and production planning. As the calculations showed, the cause of deviations is influenced by the personnel with low qualification who work with the equipment.

Thus, the performed analysis showed deviations in organization of the operative and production planning of AVTOVAZ PJSC metallurgical production, which can be eliminated as a result of the offered program of the correcting actions:
1) attestation of work places: conduct of time-study and photo of work time at the areas and shops;
2) attestation of equipment: analysis of reasons for equipment’s failure, development of the modernization plan, correction of the schedule of the preventive maintenance;
3) reconsideration of norms: verification of the organizational and normative documents of areas and implementing changes into them;
4) development of the program for increase of qualification of workers (works’ level’s correspondence to the workers’ levels).

The developed measures will allow eliminating the deviations, correcting the plans for issue of finished products, and minimizing the down time of the SPO filling line. As a result of elimination of the determined deviations, the full cost of cylinder block casting was reduced by 18.4%, which constitutes a relative yearly economy of RUB 71.8 million for manufacture of the whole volume of products at the beginning of the production process. Thus, effectiveness of the developed methodology is proves.

References


1. Tolyatti State University, Tolyatti, Russia
2. Tolyatti State University, Tolyatti, Russia
3. Tolyatti State University, Tolyatti, Russia