Multicriteria optimization model of the interaction of elements when managing network integrated structures

Modelo de criterios múltiples de optimización de las interacciones de elementos al gestionar estructuras integradas de red

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ABSTRACT:
The aim of this work is to present the results of a research study in the field of optimizing the interactions of elements in the management of integrated network structures. The study consisted in analyzing the modern developments in the field of network integrated structures management. We used sources for material flows managing and multicriteria problems solving. The study was based on the latest trends in organizational systems management. Based on the studies, a set of interrelated criteria was developed when organizing the interaction of integrated companies in network structures, which allows to quantify the cost of goods distribution, delivery time, as well as the load of equipment, storage facilities and transport. A multi-criteria model for optimizing interactions for integrated companies was developed. The model assumes that there can be an unlimited number of manufacturers and suppliers on the market.

Keywords: Network structures, optimization criteria, multicriteria model, efficiency

RESUMEN:
El objetivo de este trabajo es presentar los resultados de un estudio de investigación en el campo de la optimización de las interacciones de elementos en la gestión de estructuras de red integradas. El estudio consistió en analizar los desarrollos modernos en el campo de la gestión de estructuras integradas de red. Utilizamos fuentes para la gestión de flujos de materiales y la resolución de problemas multicriterios. El estudio se basó en las últimas tendencias en gestión de sistemas organizacionales. En base a los estudios, se desarrolló un conjunto de criterios interrelacionados al organizar la interacción de las empresas integradas en las estructuras de red, lo que permite cuantificar el costo de distribución de mercancías, el tiempo de entrega, así como la carga de equipos, instalaciones de almacenamiento y transporte. Se desarrolló un modelo de criterios múltiples para optimizar las interacciones para empresas integradas. El modelo supone que puede haber un número ilimitado de fabricantes y proveedores en el mercado.

Palabras clave: Estructuras de red, criterios de optimización, modelo multicriterio, eficiencia

1. Introduction

In practice, there are several basic organizational options for inter-company interactions. A variety of options for enterprises’ linking varies from almost complete freedom to full integration with loss of independence. The choice of a particular type of partnership is determined by the conditions in
which companies operate. Integrated structures can be divided into two main types: vertically and horizontally integrated structures (Geraskin, 2017). Regardless of the nature of the interaction, a number of common problems arise, as follows:

- each participant seeks to get the greatest profit;
- coordination of interests must be carried out at different levels;
- interaction is carried out in conditions of limited information.

Managing the interactions of companies in network structures is a complicated and knowledge-intensive process. The management requires knowledge in various fields of science (Berrah et al, 2011). We analyzed the publications of various areas related to the management of interactions of elements in network structures.

In recent decades, the material flows management was studied by Russia authors: Gadzhinsky, 2006; Anikin et al., 2006; Ryzhikov, 1969; Sterligova, 2004.

The scientific substantiation of the methods for organizational systems managing (such as contract theory, operations research, the theory of active systems, etc.) is represented in works of Germeyer, 1976; Gorelik et al., 1999; Burkov et al., 1994; Kukushkin et al., 1984; Nash et al., 2016; Aumann et al., 1988; Moulin, 1991.

The development of vector optimization and graph theory methods is presented in the works of Mashunin, 1986; Harary, 1969.

The development of solutions to multicriteria problems is presented in the works of Shikin et al., 2006; Steuer, 1986.

An analysis of publications shows that scientists have substantiated the basic principles of product management, developed many models of inventory management, but not all aspects of interaction management have been thoroughly studied and developed from a practical point of view.

It should be noted that the specific features of the interaction of companies in integrated network structures, such as the need to ensure cost reduction while reducing the delivery time of products and increasing the workload of warehousing and transport, have not yet been reflected. This circumstance determined the relevance and expediency of choosing this topic as a subject of study.

The aim of the study is to increase the efficiency of companies integrated into network structures by developing an interaction optimization model for integrated companies. Achieving this goal identified the following objectives of this research.

1. Definition of interrelated criteria for optimizing the interactions of companies integrated into network structures, which will allow to quantify the costs of product distribution, delivery times, as well as the workload of equipment, warehousing and transport.

2. Development of a multi-criteria interaction optimization model for integrated companies focused on system management methods that simultaneously takes into account the costs of product distribution, delivery times, traffic of vehicles and equipment.

The object of the research is the relations arising in the process of business and interaction of companies integrated into network structures.

The subject of the study are models that ensure the organization of effective work of integrated network structures. Information base of the research is the materials published on the research topic in the scientific, periodical and special literature.

Chapter 1 text

2. Methodology

Usually, when modeling interactions in integrated structures, many modern studies do not take into account the multifactorial nature of relationships. In our opinion, the lack of consideration of multi-criteria in theory leads to erroneous conclusions, and in practice – to the loss of profits. Therefore, to increase the efficiency of business, it is necessary to consider separately each type and form of interaction in order to establish an exhaustive number of criteria (Currarini et al, 2016).

The scientific novelty of the research is as follows.

1. Criteria for optimizing the interaction of integrated companies into network structures have been formed, which allow quantifying the distribution costs, delivery times and workload of warehousing and transport.
2. A multi-criteria model of optimization of interactions of integrated companies into network structures based on established criteria has been developed, which makes it possible to fully take into account the costs of product flow, delivery times and workload of warehousing and transport at the same time.

In the general case, when interacting elements within a single technological process, as well as when moving products along distribution channels, the following main criteria should be considered.

1. Reducing the cost of product distribution.
2. Reducing time spent on product delivery.
3. Accounting for the loading of warehousing and transport.

The article proposed a multi-criteria model that takes into account the above criteria. Vertical integration is considered without joining any alliances, etc. The interaction begins only under the condition that it is beneficial to the final part - the trading company. In practice, the individual parts of the distribution network have their own interests, but they were not taken into account in the criteria or restrictions, since the movement of a through material flow was considered. The movement of the material flow along the parts of the chain should be optimal from the point of view of the final part.

The question of whether the parts are independent economic entities remains outside the scope of the model, since the form of ownership does not matter for the model (Ivanov et al, 2018).

3. Results

There are several possible suppliers on the market for a trading company (or an assembly plant) that can supply a resource (goods, products, components) of several product groups \( A = \{1, \ldots, a, \ldots, m\} \), where \( a \) is a serial number of the nomenclature product group. \( N = \{1, \ldots, i, j \ldots n\} \) is the number of elements (workshop, production, agents, warehouses, etc.) that can participate in the interaction. Every production company \( i \in N \) can produce a resource in volume of \( 0 \leq w_i \leq W_i \) and sell the goods to another company \( j \in N, j \neq i \) in volume \( 0 \leq v_{ij} \leq V_{ij} \).

There may be a large number of suppliers for the existing trading structure. The same company can act as both a buyer and a seller.

It is required to construct a scheme for the interaction \( X \) of production and trade elements for the optimal movement of goods from producers to consumers, if the agreed supply volume is \( v_s \), purchase price of the product item from the manufacturer is \( P_{ai} \), the number of items ordered by the trading structure for one delivery is \( Q_a \), number of working days per year is \( T_r \).

The interaction of elements (Burkov et al., 2015; Geraskin, 2005) is represented as an oriented graph (Zykova, 1987), consisting of \( N \) elements (see fig. 1 and 2).

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**Figure 1**

Real interaction scheme
The company possesses the retrospective information about the values of demand in the market for previous periods $v_t$. Basing on information on the market volume, the trading company determines the required volume of deliveries $v_s$, which is subsequently subject to sale.

We introduce the following notation.
In this case, transportation and procurement costs and storage costs are denoted equally as types of costs. However, along the diagonal of the matrix, storage costs are postponed for the period, while the remaining matrix elements reflect transportation and procurement costs. Costs are calculated on the basis of cost estimates and, in turn, consist of a number of items. Transportation and manufacturing companies operate $T' \text{ days per year.}$

$T' = \text{number of working days per year, days.}$

When moving goods, costs arise:

- $c_{i,j}^d$ - the cost of shipping one batch of products from the $i$-th element to the $j$-th element, rub.;
- $c_{i,j}^h$ - storage costs of one batch of products, rub.;
- $C_{ai}$ - total costs for inventory management, rub.

The supply quantity function is as follows:

$$Q_a = f(c_{i,j}^d, c_{i,j}^h, v_s).$$  

To calculate the reorder point and the frequency of deliveries, standard models of optimal inventory and supply management are applied. That is, in this part, we do not claim to be new.

Several possible interaction schemes can be considered. When choosing a scheme, it is necessary to calculate the gain, which will occur with the optimal scheme, and compare it with the gain, which is obtained when the non-optimal scheme.

Permanent models are presented in the form of cost matrices:

$$C = \{c_{i,j}; i = 1, N; j = 1, N\}.$$  

standards for delivery of products from manufacturers to consumers based on one batch of products:

$$T = \{t_{i,j}; i = 1, N; j = 1, N\}.$$  

Load factors are:

$$K = \{k_{i,j}; i = 1, N; j = 1, N\}.$$  

All coefficients are calculated based on the agreed supply volume, that is, they show how loaded the system elements are at a certain order quantity.

The load factors of transport are determined by comparing (the ratio) the actual volume of the transported goods to the nominal capacity of transport.

The equipment load factors are determined by comparing (the ratio) of the actual volume of the processed product to the standard quantity.

The load factors for warehouse space are determined by comparing (the ratio) the actual volume of goods stored in the warehouse to the standard capacity of the warehouse.

Load factors are not permanent in the model, it is natural that with different schemes and volumes of supply they will change.

Costs include a number of parts, along with transportation and procurement costs:

$$C = \{c_{i,j}; i = 1, N; j = 1, N; i \neq j\},$$  

as well as storage costs:

$$C = \{c_{i,j}; i = 1, N; j = 1, N; i = j\}.$$  

In this case, transportation and procurement costs and storage costs are denoted equally as types of costs. However, along the diagonal of the matrix, storage costs are postponed for the period, while the remaining matrix elements reflect transportation and procurement costs. Costs are calculated on the basis of cost estimates and, in turn, consist of a number of items. Transportation
and procurement costs are expenses for a certain period (for which i orders are made), they are not necessarily related to one order. They have to be lead to the same order. 

Estimated transportation and procurement costs (per order) includes the following types of costs:

1) $g_1$ are the costs associated with the signing of the supply contract, i.e. the cost of possible travel, hospitality and negotiations, the costs associated with the need to control the process of supply, etc.;

6) $g_2$ are the cost securing the goods during transportation;

a) $g_3$ are insurance costs;

b) $g_4$ are transportation costs;

c) $g_5$ are other costs associated with the execution of the order.

It should be borne in mind that the costs of $g_2$, $g_3$, and $g_4$ are included in the transport and procurement costs only to the extent provided for by the terms of franking a cargo (free point on the way of movement of goods from the supplier to the consumer, the cost of delivery to which is included in the price of the goods).

Total transportation and procurement costs are determined by the formula:

$$\hat{n}_{i,j}^h = \frac{\sum_{b=1}^{l} g_{b}}{I}, \quad (9)$$

where $I$ is the number of orders placed and executed for a certain period;

$b$ is the cost sequence number.

Storage costs for period $T$ also include a number of items:

a) $r_1$ is loan interest;

b) $r_2$ is salaries of personnel associated with the maintenance of stocks;

c) $r_3$ is administrative expenses and utilities;

d) $r_4$ is depreciation of buildings and equipment used for stockpiling;

e) $r_5$ is protection, losses and other current expenses related to the maintenance of stocks.

The cost of storage for the period $T$ is determined by the formula:

$$\hat{n}_{i,j}^d = \frac{\sum_{b=1}^{l} r_{b}}{l}, \quad (10)$$

Naturally, the dimensions of the cost matrix elements should be the same.

The matrix of model variables is an incidence matrix. The incidence matrix of a graph is the $|N| \times |N|$ matrix, $x_{ij}$ of which is equal to one if in the column $X$ has an arc $i,j$, and zero otherwise. Incident matrix defines the structure of interaction of system elements when moving a single batch of products:

$$X = \{x_{i,j}, i = 1, N; j = 1, N\} \quad (11)$$

The incidence matrix is a mapping of a graph of supply chains consisting of a set of vertices $X = \{X_i\}$. The dimension of the incident matrix $N$ is the number of economic entities in the system. Incident matrix belongs to the space of N-dimensional vectors $X \in R^N$ (model variable space).

Three parameters are taken as optimization criteria. Cost optimization criterion:

$$F_1(X) = \sum_{i=1}^{N} \sum_{j=1}^{N} c_{i,j} \cdot x_{i,j} \rightarrow \text{min}$$

The criterion for optimizing the delivery time:

$$F_2(X) = \sum_{i=1}^{N} \sum_{j=1}^{N} t_{i,j} \cdot x_{i,j} \rightarrow \text{min}$$

Load factor optimization criteria:

$$F_3(X) = \sum_{i=1}^{N} \sum_{j=1}^{N} k_{i,j} \cdot x_{i,j} \rightarrow \text{max}$$

In the functioning of the organizational-economic system, a number of restrictions arise. We introduce the following constraint system:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} c_i \cdot x_i < v \cdot P_s.$$
The economic interpretation of constraints is that the elements of the system begin to interact with each other only when, as a result of this interaction, an economic benefit arises for the manufacturer, that is, its costs do not exceed the revenue.

The second constraint is the minimum required number of links between elements of the system, which cannot be less than the number of deliveries for the entire period:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} x_{i,j} > \frac{v_s}{Q_a}.$$ 

The third inequality imposes obligations on the manufacturer to meet the terms of delivery of products that are set by the consumer in terms of excess of supply over demand:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} t_{i,j} \cdot x_{i,j} < \frac{Q_a \cdot T^r}{v_s}.$$ 

The load factor limit is defined as follows:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} k_{i,j} \cdot x_{i,j} < \frac{2v_s}{Q_a} + 1.$$ 

Each element of the system tends to bring \(k_{i,j}\) to one (load 100% of its capacity), while \(k_{i,j} \in [0; 1]\). The value of load factors cannot be greater than the sum of the elements and the number of connections between them for the certain period. The maximum value of this criterion will correspond to the structure of the minimally related graph:

![Graph](image)

The arcs denote the load factors of the transport, and the tops denote the loading of production, warehouse or shop equipment, depending on which element of the interaction denotes the vertex. Load factors vary depending on the volume of the passing stream and the route along which the material flow is moving. The final part seeks to take into account the interests of the elements of the system (in practice, this will allow for a concession in the price of their services, etc.). In addition, elements of the system may legally and in fact belong to the final part. The restriction shows that supply volumes should coincide with the total volume of demand. The duration of the interval between deliveries coincides with the total duration of a series of consecutive periods, and the volume of supply must coincide with the aggregate volume of demand for this interval:

$$\sum_{a=1}^{m} Q_a = v_s.$$ 

Two options are possible:

1. \(v_s = v_t^{pr}\). Delivery volumes are equal to the forecast values of demand. This option is possible if the trading enterprise purchases this type of product from a single manufacturer.

2. \(v_s < v_t^{pr}\). Delivery volumes are less than expected demand. This option is usually practiced in case a merchant buys a certain type of product from several suppliers.

The following form of polynomial is used to approximate the dependence to predict the dynamic range of demand:

$$v_t^{pr} = b_0 + b_1 \cdot P_a + \frac{b_2}{2!} \cdot P_a^2 + \cdots + \frac{b_q}{q!} \cdot P_a^q = \sum_{r=0}^{q} \frac{b_r}{r!} \cdot P_a^r.$$
The formulation of performance criteria and a system of constraints makes it possible to approach the formulation of the problem of forming an interaction scheme, which consists in the following. It is required to construct a matrix of a directed graph $X$ representing the product supply structure, which contains $n$ vertices interconnected so that the selected efficiency criteria reach optimal values taking into account the constraints. The search for optimal interaction is carried out using the principles of solving multicriteria problems (Geraskin, 2017). The criteria of the task are heterogeneous, as part of the optimization criteria tends to the minimum value, and one to the maximum. The method of optimizing the supply chain and product distribution involves the choice of supply chain based on the Pareto-optimal controls (Nogin et al, 1982). The above optimization criteria are in significant economic contradiction, since with a reduction in the time of delivery of goods from producer to consumer, transport and procurement costs and costs associated with storage increase. Transportation and procurement costs of system elements increase with shorter delivery times, since in this case it is necessary to use more mobile vehicles for cargo delivery $g_4$, the associated costs increase significantly $g_1$, $g_2$, $g_3$. The cost of storing cargo also increases, due primarily to the need to maintain large quantities of products in warehouses $r_1$, $r_2$, $r_3$. In addition, each element of the system is interested in increasing load factors. However, as load factors increase, product delivery time increases.

The complexity of the problems and the inconsistency of the criteria led to the emergence of a number of mathematical models that can adequately reflect the stated problem and its multi-purpose nature (Gorodnov et al., 2005; Burkov et al, 2013). There are a number of methods for solving multicriteria problems described in (Shikin et al., 2006). The main ones include the assignment method, the ideal point method, the folding method, the constraint method, the hierarchy analysis method. These methods are general, and it is necessary to select and adapt a specific method for solving the stated problem. In solving the formulated problem, in the author’s opinion, it is advisable to use multi-criteria optimization models. In this case, the question arises about the choice of a complex optimality criterion, which includes all three criteria. We formulate the problem of vector interaction optimization in general.

Then the constraint takes the form:

$$\sum_{a=1}^{m} Q_a \leq \sum_{r=0}^{q} \frac{b_r}{r!} \cdot p_a^r.$$ 

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optf(X) = \left\{ \begin{array}{l} \max f_1(X) = \{ F_3(X), k = 1, K^+ \} \\ \min f_2(X) = \{ F_1(X), F_2(X), k = 1, K^- \} \end{array} \right\}

with the following limitations:

\[
\begin{align*}
\sum_{i=1}^{N} \sum_{j=1}^{N} c_{i,j} \cdot x_{i,j} & < \nu_s \cdot P_a \\
\sum_{i=1}^{N} \sum_{j=1}^{N} x_{i,j} & > \frac{\nu_s}{Q_a} \\
\sum_{i=1}^{N} \sum_{j=1}^{N} t_{i,j} \cdot x_{i,j} & < \frac{Q_a \cdot T^r}{\nu_s} \\
\sum_{i=1}^{N} \sum_{j=1}^{N} k_{i,j} \cdot x_{i,j} & < \frac{2\nu_s}{Q_a} + 1 \\
\sum_{a=1}^{m} Q_a & \leq \sum_{r=0}^{q} \frac{r!}{r!} \cdot P_a^r
\end{align*}
\]

where $X = \{ x_{i,j}, i = 1, N; j = 1, N \}$ is a matrix of variables;

$f_i(X)$ is vector criterion whose component aims to maximize;

$f_i(X)$ is vector criterion, each component of which is aimed at minimizing;

$K^+ \cup K^- = K$ is a set of indices of the components of the maximization and minimization criteria.

### 4. Conclusions

Analysis of the proposed model allows us to draw several conclusions. First, the model has a network nature of the interaction of elements and leads to a class of multicriteria vector optimization problems. Secondly, when forming the optimal interaction, it is advisable to use methods based on the choice of a compromise solution. Thirdly, the formation of the system takes place under conditions of limited delivery times, volumes of deliveries while simultaneously fulfilling the requirements for a minimum economic effect, the minimum required number of connections between the elements and the maximum loading of equipment, warehousing and transport. And finally, the solution to the compromise task of finding the optimal interaction is to find such a number of connections between the elements of the system and their mutual arrangement, in which the system is the least expensive in terms of costs and delivery time and most loaded in the use of equipment, storage and transport.

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